**Problem 1.** Using the definition, compute directly the simplicial homology $H_*\left(S^2; G\right)$ of the sphere $S^2$ represented as the boundary $\partial \Delta^3$ of the 3-simplex $\Delta^3$. Here $G$ is an abelian group (of coefficients for the homology).

**Problem 2.** Suppose that a simplicial complex $K$ has $n$ path components not intersecting with a subcomplex $L$. Show that $H_0(K, L; G) = nG$, the direct sum of $n$ copies of $G$.

**Problem 3.** Suppose that a simplicial complex $K \cup L$ is the union of two subcomplexes $K$ and $L$. Then, $K \cap L$, of course, is a simplicial complex, too. Show that $H_k(K, K \cap L; G) \cong H_k(K \cup L, L; G)$ for all $k$.

**Problem 4.** Given a commutative diagram

\[
\begin{array}{cccccc}
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
\downarrow^{\alpha} & & \downarrow^{\beta} & & \downarrow^{\gamma} & & \\
0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0
\end{array}
\]

of abelian groups with exact rows, prove that there is an exact sequence

\[
0 \rightarrow \text{Ker} \alpha \rightarrow \text{Ker} \beta \rightarrow \text{Ker} \gamma \rightarrow \text{Coker} \alpha \rightarrow \text{Coker} \beta \rightarrow \text{Coker} \gamma \rightarrow 0.
\]

**[Hint: Instead of doing it from scratch, you may use general procedures generating long exact sequences. In other words, you may use Theorem 2.16 from Hatcher.]**

**Problem 5.** Thinking of the real projective plane $\mathbb{R}P^2$ as the sphere with antipodal points identified or, equivalently, the disk with antipodal points of its boundary identified, take an abstract simplicial complex (not just a $\Delta$-complex!) (whose geometric realization is) homeomorphic to $\mathbb{R}P^2$. Do not prove that this is the case, but rather compute the homology $H_*\left(\mathbb{R}P^2; \mathbb{Z}\right)$ and $H_*\left(\mathbb{R}P^2; \mathbb{Z}_2\right)$ by analyzing the simplicial chain complexes. **[Hint: There should be quite a few simplices, we should have an example of a simplicial representation of $\mathbb{R}P^2$ in class with ten 2-simplices.]**