Problem 1. Show that if one of the differentials $d'$ and $d''$ in a double complex vanishes, then the spectral sequence collapses at $E^2$.

Problem 2. Prove that if at least one of simplicial sets $K_\bullet, L_\bullet$ has finite type (i.e., finitely many nondegenerate elements in each degree), then $|K \times L|$ is homeomorphic to $|K| \times |L|$.

Problem 3. Show that if two simplicial maps $f, g : K \to L$ are homotopic ($f \sim g$, if there exists a simplicial map $H : K \times I \to L$, where $I = \Delta[1]$, such that $H_0 = f$ and $H_1 = g$), then $|f| \sim |g| : |K| \to |L|$. [Here $\Delta[1]$ is the simplicial set defined as the contravariant functor $\text{Mor}_\Delta(\Delta, \{1\})$ from the category $\Delta$ to the category of sets.]

Problem 4. Show that the bijections $\phi : SS(K_\bullet, \text{Sing}_\bullet(X)) \to \text{Top}(|K|, X)$ and $\psi : \text{Top}(|K|, X) \to SS(K_\bullet, \text{Sing}_\bullet(X))$ preserve homotopies.

Problem 5. Let $G$ be a topological group, $BG$ its classifying space, and $\Omega BG$ the based loop space of $BG$. Show that there exists a weak equivalence: $G \sim \Omega BG$.

Problem 6. Show explicitly, using the Milnor construction of a classifying space, that $B\mathbb{Z}_2 = \mathbb{R}P^\infty$ and $BS^1 = \mathbb{C}P^\infty$. 

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