Homework Assignment 1 Due Thursday 1/28/2021.

The first quiz will be held on this date, on the material of the exercises listed below, both the exercises to be handed in and the further exercises listed at the end.

The vast majority of exercises in Edwards and Penney have solutions given at the end of the book. To get round this, I am specifying questions for you to hand in that are similar to questions in the book, but not the same, so no answer is given. If you want to make sure that you can do a question of a certain kind and know that you got the right answer, you can do the exercises in the book, and check the answer at the end.

My intention is that you should do all of the exercises specified below, both the ones typed out below and also the questions in the list at the end. To make life easier, I am only requiring that you hand in a part of this total list of questions, namely the ones typed below.

Upload to Gradescope your solutions to the following:

1a (1.1 qn 30) Write a differential equation of the form \( \frac{dy}{dx} = f(x, y) \) having the function \( g \) as one of its solutions, where \( y = g(x) \) is described by the following geometric property of its graph:

the graph of \( g \) is normal to every curve of the form \( y = x^2 + k \) (\( k \) is a constant) where they meet.

1b (like 1.2 qn 25) The brakes of a car are applied when it is moving at 80 km/h and provide a constant deceleration of 16 meters per second per second (m/s\(^2\)). How far does the car travel before coming to a stop?

1c (like 1.2 qn 27) A ball is thrown straight downward from the top of a tall building. The initial speed of the ball is 5 m/s. It strikes the ground with a speed of 50 m/s. How tall is the building?

1d (like 1.2 qn 36) Suppose a woman has enough “spring” in her legs to jump (on earth) from the ground to a height of 2.5 feet. If she jumps straight upward with the same initial velocity on the moon – where the surface gravitational acceleration is (approximately) 8 ft/s\(^2\) – how high above the surface will she rise?

1e (like 1.3 qn 22) First use the method of Section 1.3 Example 2 to construct a slope field for the differential equation \( y' = y - x \). Then sketch the solution curve corresponding to the initial condition \( y(0) = -3 \). Finally, use this solution curve to estimate the value \( y(-2) \) of this solution at \( x = -2 \).

1f (like 1.3 qn 28) Verify that if \( C \) is a constant, then the function

\[
y(x) = \frac{-1}{C + \ln x}
\]

satisfies the differential equation \( xy' = y^2 \) for all \( x \). Construct a slope field and several of the solution curves. Then determine (in terms of \( a \) and \( b \)) how many different solutions the initial value problem \( xy' = y^2, \ y(a) = b \) has — one, none, or infinitely many.
1g (like 1.4 qn 26) Find an explicit particular solution of the initial value problem

\[ \frac{dy}{dx} = 3x^2y^2 - 4x^3y^2, \quad y(1) = -1 \]

1h (like 1.4 qn 33) A certain city had a population of 24,000 in 1980 and 32,000 in 1990. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2030?

Further exercises from chapter 1 of Edwards and Penney (not to be uploaded to Gradescope):
1.1: 3, 15, 17, 19, 28
1.2: 3, 11
1.3: 1
1.4: 1, 5, 23, 27, 49