GG. Let $G$ be the group of all isometries of the cube, and let $H$ be the subgroup consisting of rotations which preserve the cube. Let $-1$ denote the element of $G$ which is the transformation of $\mathbb{R}^3$ given by multiplication by $-1$.

(a) Show that $G = H \times \langle -1 \rangle$.

(b) Show that if $g \in G$ is any element other than $-1$ then $G \neq H \times \langle g \rangle$.

(To do this you may need to prove that the center of $H$ is $\{e\}$. Either use the isomorphism with $S_4$ or note that if you conjugate one rotation by another rotation you get rotation about an axis obtained by applying the second rotation to the axis of the first.)

HH*. (a) Let $G$ be the group of all isometries of the tetrahedron, and let $H$ be the subgroup consisting of rotations which preserve the tetrahedron. Determine whether or not $G = H \times K$ for some subgroup $K$ of $G$.

(b) Let $G$ be the group of all isometries of the icosahedron, and let $H$ be the subgroup consisting of rotations which preserve the icosahedron. Determine whether or not $G = H \times K$ for some subgroup $K$ of $G$. 