1. Show that the two extensions $0 \to \mathbb{Z}^\mu \to \mathbb{Z} \to \mathbb{Z}/3\mathbb{Z} \to 0$ and $0 \to \mathbb{Z}^\mu' \to \mathbb{Z} \to \mathbb{Z}/3\mathbb{Z} \to 0$ are not equivalent, where $\mu = \mu'$ is multiplication by 3, $\epsilon(1) \equiv 1 \pmod{3}$ and $\epsilon'(1) \equiv 2 \pmod{3}$.

2. (D&F 10.4, 4) Show that $\mathbb{Q} \otimes \mathbb{Z} \mathbb{Q}$ and $\mathbb{Q} \otimes \mathbb{Q} \mathbb{Q}$ are isomorphic left $\mathbb{Q}$-modules. [Show they are both 1-dimensional vector spaces over $\mathbb{Q}$.]

3. (D&F 10.4, 5) Let $A$ be a finite abelian group of order $n$ and let $p^k$ be the largest power of the prime $p$ dividing $n$. Prove that $\mathbb{Z}/p^k\mathbb{Z} \otimes_\mathbb{Z} A$ is isomorphic to the Sylow $p$-subgroup of $A$.

4. (D&F 10.4, 6) If $R$ is any integral domain with quotient field $\mathbb{Q}$, prove that

$$(\mathbb{Q}/R) \otimes_R (\mathbb{Q}/R) = 0.$$

5. (D&F 10.4, 11) Let $\{e_1, e_2\}$ be a basis of $V = \mathbb{R}^2$. Show that the element $e_1 \otimes e_2 + e_2 \otimes e_1$ in $V \otimes_\mathbb{R} V$ cannot be written as a simple tensor $v \otimes w$ for any $v, w \in \mathbb{R}^2$.

6. Show that, as a ring, $\mathbb{Q}(\sqrt{2}) \otimes_\mathbb{Q} \mathbb{Q}(\sqrt{2})$ is the direct sum of two fields. [The ring multiplication is $(a \otimes b)(c \otimes d) := ac \otimes bd$ on basic tensors. See Proposition 19 of D&F. Assume question 25 from 10.4 of D&F.]

7. (D&F 10.5, 14(a)) Let $0 \to L \xrightarrow{\psi} M \xrightarrow{\phi} N \to 0$ be a sequence of $R$-modules.

(a) Prove that the associated sequence

$$0 \to \text{Hom}_R(D, L) \xrightarrow{\psi^*} \text{Hom}_R(D, M) \xrightarrow{\phi^*} \text{Hom}_R(D, N) \to 0$$

is a short exact sequence of abelian groups for all $R$-modules $D$ if and only if the original sequence is a split short exact sequence. [Assume that Hom is left exact: do not prove this. To show the sequence splits, take $D = N$ and show the lift of the identity map in $\text{Hom}_R(N, N)$ to $\text{Hom}_R(N, M)$ is a splitting homomorphism for $\phi$.]

(b) Do not bother with this part of the question. It is a similar statement obtained by applying $\text{Hom}_R(-, D)$ to the short exact sequence.