Math 3593 Practice for exam 1.

The exam on February 15 will be on what we have studied in Sections 2.10, 3.1, 3.1, 3.2, 3.3, and 3.4. The topics are the inverse and implicit function theorems, the various definitions of a manifold and showing that certain sets are or are not manifolds, tangent spaces and Taylor polynomials. On this exam you may use a calculator, but you may not use the book or notes.

1. Find the equation of the line tangent to the curve with parametrization \( \phi(t) = (t^3, \sin t, e^t)^T \) at the point \((1, \sin(1), e)^T\).
2. Find the equation of the tangent plane to the graph of \(\sin(x + y)\) at \((\pi, \pi/2)^T\).
3. Find the equation of the tangent plane to \(x^2 + 2y^2 + 3z^2 = 6\) at \((1, 1, 1)^T\).
4. Find the equation of the tangent line to the intersection of \(x^2 + 2y^2 + 3z^2 = 6\) and \(x^2 + y^2 - z^2 = 1\) at the point \( \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \).

In the above questions, calculate also the tangent vector space to the manifold, at the point mentioned. Make sure you can describe it in all ways: parametrically, as the set of points satisfying a system of linear equations, and by finding a basis for it.

The questions above and below summarize the things we have been doing. Some of the questions below were set on homework. The intention is to give you something to look through for review. It would not make sense to do them all.

Section 2.10: nos 2, 4, 6, 7, 8, 9, 10, 12, 16
Section 2.11: nos 2.31, 2.32, 2.33
Section 3.1: nos 7, 8, 21
Section 3.3: nos 12, 13
Section 3.4: nos 1, 3,
Section 3.10: nos 3.1, 3.2, 3.5, 3.7, 3.10, 3.11.