Math 8201 **Homework 8** **PJW**

Date due: November 7, 2005. There will be a quiz on this date.

Hand in only the starred questions.

Section 4.5 8, 10, 11, 14, 15, 16, 17*, 18, 21, 24*, 30, 32, 33*, 34, 35, 41, 42*

There are many questions on Sylow's theorem in the preliminary written exams from past years. For example, the question which appeared as question 5 in the Spring of 1994 is challenging:

5. (Spring 1994)
   (i) Let G be a finite group. Prove that the number of conjugates in G of a subgroup H equals the index of its normalizer $N_G(H)$ in G.
   (ii) Let now G be a simple group of order $1092 = 4 \cdot 3 \cdot 7 \cdot 13$.
   a) Find the number of Sylow 13-subgroups and the number of Sylow 7-subgroups of G.
   b) Prove that G has a single conjugacy class of subgroups of index 14.
   c) Prove that G has no subgroup of index 13.
   [You may assume Sylow's theorems.]

BB*. Let G be a finite group and let $O_p(G)$ denote the unique largest normal p-subgroup of G, which contains all other normal p-subgroups of G and whose existence was established in question P of sheet 4. Show that $O_p(G)$ equals the intersection of all the Sylow p-subgroups of G:

$$O_p(G) = \bigcap_{P \text{ a Sylow } p-\text{subgroup}} P.$$

[You do not have to do question P. Show that the group on the right contains every normal p-subgroup of G, and is a normal p-subgroup of G.]

CC. Let G be a finite group and H a subgroup. Let P_H be a Sylow p-subgroup of H. Prove that there exists a Sylow p-subgroup P of G such that $P_H = P \cap H$.

DD. Prove that one of the Sylow subroups of a group of order 40 is normal. Give an explicit example of a group of order 40 in which the Sylow 2-subgroup is not normal.

Section 4.6 2*, 3, 4, 5, 6