Math 5594 **Homework 3, due Monday October 2, 2006** **PJW**

1. (page 36 no. 2 + an extra sentence) Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/3} \end{pmatrix},\ C = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

and let $G = \langle a : a^3 = 1 \rangle \cong C_3$. Show that each of the functions $\rho_j : G \to GL(2,\mathbb{C})$ $(1 \leq j \leq 3)$, defined by

$$\begin{aligned} \rho_1 &: a^r \to A^r, \\ \rho_2 &: a^r \to B^r, \\ \rho_3 &: a^r \to C^r \quad (0 \leq r \leq 2), \end{aligned}$$

is a representation of G over $\mathbb{C}$. Which of these representations are faithful? Construct a further representation in which $\rho_4(a)$ is neither a diagonal matrix nor a power of C.

2. (page 36 no. 5) Let $G = D_{12} = \langle a, b : a^6 = b^2 = 1,\ b^{-1}ab = a^{-1} \rangle$. Define the matrices A, B, C, D over $\mathbb{C}$ by

$$A = \begin{pmatrix} e^{i\pi/3} & 0 \\ 0 & e^{-i\pi/3} \end{pmatrix},\ B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$C = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix},\ D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Prove that each of the functions $\rho_k : G \to GL(2,\mathbb{C})$ $(k = 1, 2, 3, 4)$, given by

$$\begin{aligned} \rho_1 &: a^r b^s \to A^r B^s, \\ \rho_2 &: a^r b^s \to A^{3r} (-B)^s, \\ \rho_3 &: a^r b^s \to (-A)^r B^s, \\ \rho_4 &: a^r b^s \to C^r D^s \quad (0 \leq r \leq 5,\ 0 \leq s \leq 1), \end{aligned}$$

is a representation of G. Which of these representations are faithful? Which are equivalent?

3. (page 37 no. 6) Give an example of a faithful representation of D_8 of degree 3.

4. (page 37 no. 7) Suppose that ρ is a representation of G of degree 1. Prove that $G/\operatorname{Ker}\rho$ is abelian.

5. (page 37 no. 8) Let ρ be a representation of the group G. Suppose that g and h are elements of G such that $(g\rho)(h\rho) = (h\rho)(g\rho)$. Des it follow that $gh = hg$?

6. (Only if we get that far! Page 52 no. 3) Which of the four representations of D_{12} defined in Exercise 2 are irreducible?