3593 Exam 2 practice questions
Sections of the book to be tested: 3.5, 3.6, 3.7, 4.1, 4.3, 4.5 and 4.8

1. Prove that a subset of a set of volume zero has volume zero.

2. Consider the functions defined on \( \mathbb{R}^2 \)

\[
f(x, y) = \begin{cases} 
1 & \text{if } y = x^2 \text{ and } -1 \leq x \leq 1 \text{ is rational}, \\
0 & \text{otherwise}.
\end{cases}
\]

\[
g(x, y) = \begin{cases} 
x y & \text{if both } x \text{ and } y \text{ are rational between } -1 \text{ and } 1, \\
0 & \text{otherwise}.
\end{cases}
\]

Do \( \int_{\mathbb{R}^2} |d^2x| \) and \( \int_{\mathbb{R}^2} |d^2x| \) exist? If so, what are their values?

3. Let \( \sigma \) be the permutation \( \sigma(1) = 2, \sigma(2) = 3, \sigma(3) = 4, \sigma(4) = 1 \) and let \( \tau \) be the permutation \( \tau(1) = 1, \tau(2) = 4, \tau(3) = 3, \tau(4) = 2 \). What is the sign of the permutation \( \sigma \tau \)?

Relevant questions from the book:
Section 3.5 page 341: questions from assignment 4 and
Section 3.6 page 349: questions from assignment 4 and the questions listed below from Section 3.9.
Section 3.7 page 366: questions from assignment 5 and nos 21, 22 from Section 3.9 below.
Section 3.9 page 386: 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 25
Section 4.1 page 405: 10, 14, 15
Section 4.3 page 427: 5
Section 4.5 page 445: 7, 8, 11, 12, 14, 15, 16, 18
Section 4.8 page 474: 2, 12, 13, 15
Section 4.12 page 514: 11, 12, 13