Math 8201 **Homework 1** **PJW**

Date due: September 15, 2008

Hand in only the six starred questions.

Section 2.2 nos. 2.1, 2.7(1), 2.15*

Section 2.3 nos. 2.20*, 2.23, 2.24 (Show also that if we remove the hypothesis that p be a prime divisor of k, and replace it simply by the hypothesis that p is a prime, then the result is no longer true), 2.25, 2.26*, 2.27, 2.28*

Section 2.4 nos. 2.31, 2.32, 2.34, 2.37*, 2.38

A If x is an element of finite order n in a group G, prove that the elements

$$1, x, x^2, \ldots, x^{n-1}$$

are all distinct.

B If x is an element of infinite order in a group G, prove that the elements x^n, $n \in \mathbb{Z}$ are all distinct.

C* If $n = 2k$ is even and $n \geq 4$ show that $z = \rho^k$ is an element of order 2 which commutes with all elements of D_{2n}. Show also that z is the only nonidentity element of D_{2n} which commutes with all elements of D_{2n}.

D If n is odd and $n \geq 3$, show that the identity is the only element of D_{2n} which commutes with all elements of D_{2n}.

E Let G be the group of symmetries in $\mathbb{R}^3$ of a tetrahedron. Show that $|G| = 24$.

F Let G be the group of symmetries in $\mathbb{R}^3$ of a cube. Show that $|G| = 48$.

G Let G be the group of v in $\mathbb{R}^3$ of an icosahedron. Show that $|G| = 120$.