Math 8201 **Homework 10** **PJW**

Date due: November 17, 2008. There will be a quiz on this date. Hand in the 4 starred questions.

Section 3.6

3.58, 3.60*, 3.63, 3.64*, 3.66*

XX Find the g.c.d and the l.c.m in $\mathbb{Z}[i]$ of 85 and 1+13i. Find the g.c.d and the l.c.m in $\mathbb{Z}[i]$ of $47 - 13i$ and $53 + 56i$.

YY* (a) Prove that $\mathbb{Z}[\sqrt{2}] := \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ is a Euclidean domain with respect to $N(a + b\sqrt{2}) = |a^2 - 2b^2|$, showing that N is a (multiplicative) norm.
(b)Show that there are infinitely many units in $\mathbb{Z}[\sqrt{2}]$.
(c)Find a pair of integers a and b, both larger than 100, for which $a^2 - 2b^2 = 1$.
(d)Find the g.c.d in $\mathbb{Z}[\sqrt{2}]$ of $1 + 5\sqrt{2}$ and $2 + 3\sqrt{2}$.
(e)Express $1 + 5\sqrt{2}$ as a product of irreducible elements of $\mathbb{Z}[\sqrt{2}]$, proving that the elements in the product are indeed irreducible.

ZZ (a) How many essentially different ways are there to write $29 \cdot 37$ as a sum of square of two integers? We regard $a^2 + b^2 = b^2 + a^2 = (-a)^2 + b^2$ etc as 'the same'.
(b) How many essentially different ways are there to write $29 \cdot 31$ as a sum of square of two integers?
(c) How many incongruent right-angled triangles are there with hypotenuse of length $17^2 = 289$ and sides of integer lengths? (Only consider triangles with non-zero area.)