Math 8201
Homework 11

Date due: Wednesday November 26, 2008. Hand in the 6 starred questions.

ZZ (a) How many essentially different ways are there to write $29 \cdot 37$ as a sum of squares of two integers? We regard $a^2 + b^2 = b^2 + a^2 = (-a)^2 + b^2$ etc as ‘the same’.
(b) How many essentially different ways are there to write $29 \cdot 31$ as a sum of squares of two integers?
(c) How many incongruent right-angled triangles are there with hypotenuse of length $17^2 = 289$ and sides of integer lengths? (Only consider triangles with non-zero area.)

Section 3.8 3.82, 3.83

AAA* For every commutative ring $R$, prove that $R[x]/(x) \cong R$. Prove also that

$$R[x]/(x+1) \cong R[x]/(x-1).$$

For the moment we will only do Theorems 3.110-3.112 from section 3.8.

Section 6.1 6.1, 6.3, 6.5, 6.6*, 6.9, 6.11, 6.12*, 6.16*

BBB* Let $R$ be a finite commutative ring with identity. Prove that every prime ideal of $R$ is a maximal ideal.

CCC* Let $R$ be a (not necessarily commutative) ring whose only left ideals of $(0)$ and $R$. show that $R$ is a division ring.

DDD Let $I$ and $J$ be ideals of a commutative ring $R$ and assume $P$ is a prime ideal of $R$ that contains $I \cap J$. Prove that either $I$ or $J$ is contained in $P$.

EEE Let $R$ be a commutative ring and suppose for each $a \in R$ there is a positive integer $n$ (depending on $a$) such that $a^n = a$. Prove that every prime ideal of $R$ is a maximal ideal.