As well as Section 5.3, pages 212 - 215 from Section 4.1 about solvable groups are relevant this week.

Date due: October 27, 2008 Hand in the 5 starred questions.

Section 5.3 5.33, 5.34*, 5.39*, 5.40, 5.43*, 5.44, 5.45*, 5.46,

MM Calculate the commutator subgroup $G'$ (in particular determining its order) when $G = D_{10}$ and $G = D_{12}$. List all composition series for these groups.

NN The definition of a characteristic subgroup is given in Exercise 5.19 on page 277, which also describes a property of the commutator subgroup. Show that if $H \leq K \triangleleft G$ are subgroups and if $H$ is a characteristic subgroup of $K$ then $H \triangleleft G$.

OO* Let $G$ be a solvable group.

(i) Prove that if $H$ is a nontrivial normal subgroup of $G$ then there is a nontrivial subgroup $A$ of $H$ with $A \triangleleft G$ and $A$ abelian.

(ii) Show that $G$ has a chain of subgroups $1 = N_0 \leq N_1 \leq \cdots \leq N_t = G$ for which $N_i \triangleleft G$ and $N_i/N_{i-1}$ is abelian for all $i$.

(iii) Show that if $G$ is finite then every minimal normal subgroup of $G$ is abelian.

(iv) Deduce that if $G$ is finite and $K$ is a minimal normal subgroup of $G$ then $K$ is a $p$-group for some prime $p$ and that $x^p = e$ for every $x \in k$. (Such a group $K$ is a vector space over the field with $p$ elements and so is in fact a direct product of cyclic groups of order $p$.)