Math 3592 review for exam 1

On the exam points in \( \mathbb{R}^3 \) are sometimes written as (for example) \((1, 2, 3)\), rather than \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \). This sheet is supposed to help you by providing some extra practice. It should not be interpreted that what is on this sheet is the only practice you should do, or that questions like the ones here are the only kind you will be asked. You are advised to review the material we have covered more broadly than what is on this sheet.

1. Let \( L_1 \) be the line passing through the point \((1, 2, 3)\) perpendicular to the plane \( P_1 \) with equation \( 3x - y + 2z = 4 \). Find equations for \( L_1 \) in the form

\[
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
\]

2. Let \( L_2 \) be the line \( x = y = z \). Find the shortest distance from \( L_1 \) to \( L_2 \).

3. Find the point of intersection of \( L_2 \) and \( P_1 \).

4. Find the equations of the line parallel to \( L_2 \) which passes through \((10, 1, -2)\).

5. Find the shortest distance from the point \((5, 1, 3)\) to \( P_1 \).

6. Find the shortest distance from the point \((5, 1, 3)\) to \( L_1 \).

7. Does there exist a matrix \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) with

\[
T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}.
\]

If so, find the matrix of such a \( T \).

8. Let \( f : \mathbb{R}^m \rightarrow \mathbb{R}^n \) be a linear mapping with matrix \( T \). Which, if any, of the following statements are logically equivalent to each other? (They are logically equivalent if they mean exactly the same thing.)

   (a) \( T \) has a right inverse.
   (b) \( T \) has a left inverse.
   (c) \( T \) is invertible.
   (d) \( f \) is one-to-one.
   (e) \( f \) is onto.
   (f) \( f \) is one-to-one and onto.

9. Can you find a right inverse to the matrix \( T = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \)? What about a left inverse? Is \( T \) the matrix of a one-to-one mapping? An onto mapping?