Math 3592H **Honors Mathematics I Fall Semester 2012**

Assignment 8 - Due Thursday 11/01/2012

Read: Hubbard and Hubbard Section 1.9. I hope to start on 2.1 during the week.

Exercises:
Hand in only the exercises which have stars by them.

Section 1.9 (page 154): 1.9.2c*
Section 1.10 (pages 155-160): 26b, 27, 28*, 29*, 30, 31

Extra Questions:

1. Given that $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 + xy + 1 \\ y^2 + 2 \end{pmatrix}$, $\quad g\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u + v \\ 2u \\ v^2 \end{pmatrix}$ find the derivative matrix of the composite function $g \circ f$ at (x,y) = (1,1); also at (x,y) = (0,0).

2. Consider the curve defined parametrically by $f(t) = \begin{pmatrix} t \\ t^2 - 4 \\ e^{t-2} \end{pmatrix}$, $\quad -\infty < t < \infty$. Let g(x,y,z) be a real-valued differentiable function of three variables. If a = (2,0,1) and $\frac{\partial g}{\partial x}(a) = 4$, $\frac{\partial g}{\partial y}(a) = 2$, $\frac{\partial g}{\partial z}(a) = 2$, find $\frac{d(g \circ f)}{dt}$ at t = 2.

3*. Let $z = xy^2$ and suppose that x = 2u + 3v. Assume also that y is a function of u and v with the properties that when (u,v) = (2,1) then $y = -1$, $\frac{\partial y}{\partial u} = 5$ and $\frac{\partial y}{\partial v} = -2$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ when (u,v) = (2,1).

4*. Show that for a differentiable real-valued function g(x,y),

$$\frac{d\mathrm{g}(x, x)}{dx} = \frac{\partial \mathrm{g}}{\partial x}(x, x) + \frac{\partial \mathrm{g}}{\partial y}(x, x).$$

Apply this equation to the function $g(x, y) = x^y$ and hence compute the derivative of the function $u(x) = x^x$. [Hint: Consider the composite of g with the function f(x) = (x,x).]

Comments:
The main result in this section, Theorem 1.9.8, is important to know, but I do not think we need trouble ourselves much with the proof. The proof depends on Theorem 1.9.1 and here also we don't need to bother. You will not be tested on those proofs. What is more important in section 1.9 is to see the type of strange behavior that can occur in the examples.