Worksheet on the definition of a limit

The definition of \( f(x) \to L \) as \( x \to a \) is: for each \( \epsilon > 0 \) there exists a number \( \delta > 0 \) so that \( |f(x) - L| < \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).

A. Which (if any) of the following means the same thing as \( f(x) \) does not tend to \( L \) as \( x \) tends to \( a \)?

1. For each choice of \( \epsilon > 0 \) there exists a \( \delta > 0 \) so that \( |f(x) - L| > \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).
2. For each choice of \( \epsilon > 0 \) there exists a \( \delta > 0 \) so that \( |f(x) - L| > \epsilon \) for some \( x \) with \( 0 < |x - a| < \delta \).
3. For some choice of \( \epsilon > 0 \) and for every choice of \( \delta > 0 \) we have \( |f(x) - L| > \epsilon \) for some \( x \) with \( 0 < |x - a| < \delta \).
4. For some choice of \( \epsilon > 0 \) and for every choice of \( \delta > 0 \) we have \( |f(x) - L| > \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).
5. For some choice of \( \epsilon > 0 \) there exists a \( \delta > 0 \) so that \( |f(x) - L| > \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).
6. There exists a number \( M \neq L \) so that for each choice of \( \epsilon > 0 \) there exists a number \( \delta > 0 \) so that \( |f(x) - M| < \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).
7. There exists a number \( \delta > 0 \) so that for each choice of \( \epsilon > 0 \), \( |f(x) - L| > \epsilon \) for every \( x \) with \( 0 < |x - a| < \delta \).

B. Which of the following means \( f(x) \to \infty \) as \( x \to L \), and which means \( f(x) \to L \) as \( x \to \infty \)?

1. For every choice of number \( \epsilon \) there exists a number \( N \) so that \( |f(x) - L| < \epsilon \) for every \( x \) with \( x > N \).
2. For every choice of number \( N \) there exists \( \delta > 0 \) so that \( |f(x)| > N \) for every \( x \) with \( 0 < |x - L| < \delta \).
3. For every choice of number \( N \) there exists \( \delta > 0 \) so that \( f(x) > N \) for every \( x \) with \( 0 < |x - L| < \delta \).
4. For every choice of number \( N \) there exists \( \delta > 0 \) so that \( |f(x) - L| < \delta \) for every \( x \) with \( x > N \).