Worksheet on the definition of a limit

The definition of $f(x) \to L$ as $x \to a$ is: for each $\epsilon > 0$ there exists a number $\delta > 0$ so that $|f(x) - L| < \epsilon$ for every x with $0 < |x - a| < \delta$.

A. Which (if any) of the following means the same thing as $f(x)$ does **not** tend to L as x tends to a?

1. For each choice of $\epsilon > 0$ there exists a $\delta > 0$ so that $|f(x) - L| > \epsilon$ for every x with $0 < |x - a| < \delta$.
2. For each choice of $\epsilon > 0$ there exists a $\delta > 0$ so that $|f(x) - L| > \epsilon$ for some x with $0 < |x - a| < \delta$.
3. For some choice of $\epsilon > 0$ and for every choice of $\delta > 0$ we have $|f(x) - L| > \epsilon$ for some x with $0 < |x - a| < \delta$.
4. For some choice of $\epsilon > 0$ and for every choice of $\delta > 0$ we have $|f(x) - L| > \epsilon$ for every x with $0 < |x - a| < \delta$.
5. For some choice of $\epsilon > 0$ there exists a $\delta > 0$ so that $|f(x) - L| > \epsilon$ for every x with $0 < |x - a| < \delta$.
6. There exists a number $M \neq L$ so that for each choice of $\epsilon > 0$ there exists a number $\delta > 0$ so that $|f(x) - M| < \epsilon$ for every x with $0 < |x - a| < \delta$.
7. There exists a number $\delta > 0$ so that for each choice of $\epsilon > 0$, $|f(x) - L| > \epsilon$ for every x with $0 < |x - a| < \delta$.

B. Which of the following means $f(x) \to \infty$ as $x \to L$, and which means $f(x) \to L$ as $x \to \infty$?

1. For every choice of number ϵ there exists a number N so that $|f(x) - L| < \epsilon$ for every x with $x > N$.
2. For every choice of number N there exists $\delta > 0$ so that $|f(x)| > N$ for every x with $0 < |x - L| < \delta$.
3. For every choice of number N there exists $\delta > 0$ so that $f(x) > N$ for every x with $0 < |x - L| < \delta$.
4. For every choice of number N there exists $\delta > 0$ so that $|f(x) - L| < \delta$ for every x with $x > N$.