Math 8300 Topics in Algebra Spring Semester 2017

Representations of the symmetric and general linear groups

Instructor

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Office Hours: MWF 11:15 - 12:05 or by appointment.

Syllabus

Simple modules, Specht modules and Young modules for the symmetric groups over arbitrary fields.
The Schur algebras. Polynomial representations of GL_n. Schur-Weyl duality. Highest weight categories and quasi-hereditary algebras. q-versions.
Weight-theoretic description of simple modules for GL_n. The field of definition is a splitting field. Steinberg's tensor product theorem.
The Dickson invariants.
Functors from vector spaces to vector spaces: generic representation theory. Description of the simple functors, projectives and injectives.
Representation stability and FI-modules.

Course Content and Goals

The main focus will be on representations of the symmetric and general linear groups over fields of positive characteristic. Here, the defining characteristic of the general linear groups and of their representations will be the same. The representation theory of the symmetric groups in this situation is more delicate and richer than in characteristic zero, but in some ways the representations of general linear groups are easier in the defining characteristic. We will study the modules for symmetric groups that are closely related to permutation modules defined by Young subgroups, parametrizing their summands (the Young modules), developing the tools to understand the submodule structure of these modules, and giving the parametrization of simple modules due to James. Schur-Weyl duality provides a powerful method to relate representations of symmetric groups to general linear groups, and vice-versa. The representations of the general linear group that arise are exactly those of the Schur algebra, and these are the polynomial representations. We show this, and establish their basic properties. We will indicate what is involved in showing that the Schur algebras are quasi-hereditary, and develop the properties of such algebras and their highest-weight module categories. After some other fundamental properties of representations of general linear groups we will study the category of functors from vector spaces to vector spaces. The simple functors are parametrized by selecting a dimension, and a simple representation of the general linear group in that dimension, and all their evaluations are simple representations of different general linear groups. This provides a way to relate representations of different general linear groups to each other. We conclude with a discussion of recent work on functors on the category of finite sets with injective maps that relate to representation stability.

A few texts

GD James, The representation theory of the symmetric groups, Lecture notes in mathematics 682, Springer-Verlag 1978.
JA Green, Polynomial representations of GL_n, Lecture notes in mathematics 830, Springer-Verlag, 2nd ed. 2007. There is online access via the library.
S Martin, Schur algebras and representation theory, Cambridge tracts in mathematics 112, Cambridge 1993.
A. Mathas, Iwahori-Hecke algebras and Schur algebras of the symmetric group, University lecture series 15, AMS 1999.
V Dlab, Quasi-hereditary algebras, appendix in: Drozd and Kirichenko, Finite dimensional algebras, Springer-Verlag 1994.
M. Cabanes and M. Enguehard, Representation theory of finite reductive groups, CUP 2004.
NJ Kuhn, The generic representation theory of finite fields: a survey of basic structure, in: eds. H Krause and CM Ringel, Infinite Length Modules, Birkhauser 2000 (now Springer), 193-212.

Course Assessment

I will assign a set of homework problems roughly every 2 weeks, giving a total of six homework assignments altogether. If you make a genuine attempt at question parts scoring at least 50% of the total you will get an A for the course. You do not have to obtain correct solutions to these questions, only make genuine attempts (in my opinion). I believe that it is extremely difficult to obtain a sound and permanently lasting command of the material presented without doing some work which actively involves the student. It should be possible for everyone who wishes to obtain an A on this course.

Expectations of written work

Most of the time in the conventional homework problems, to satisfy my criterion of making a genuine attempt you will need to write down explanations for the calculations and arguments you make. Where explanations need to be given, these should be written out in sentences i.e. with verbs, capital letters at the beginning, periods at the end, etc. and not in an abbreviated form.
I encourage you to form study groups. However everything to be handed in must be written up in your own words. If two students hand in identical assignments, they will both receive no credit.

Prerequisites

The content of the Math 8200 algebra sequence is sufficient as a prerequisite. The material from Math 8300 last semester is not needed this semester, although the idea of presenting this choice of material is to fill out the picture of representations of general linear groups that was presented last semester. The experience gained from the last semester may be useful in providing orientation and motivation for what we do.

Incompletes

These will only be given in exceptional circumstances. A student must have satisfactorily completed all but a small portion of the work in the course, have a compelling reason for the incomplete, and must make prior arrangements with me for how the incomplete will be removed, well before the end of the quarter.

Date of this version of the schedule: 1/14/2017