Math 3592 review for exam 3

These are just some of the types of question that might appear on the exam. You should review the material we have covered more broadly than what is on this sheet. You will be tested on the material in Sections 2.1 - 2.6 that we have studied, including things like the extra questions on the homework sheets. There are many computational things, to do with interpreting pivots in the echelon form to test for independence of vectors, whether vectors span, etc, that you should be on top of.

1. Find a basis for the vector space which is the intersection in $\mathbb{R}^4$ of the hyperplanes $w+x+y+z=0$ and $w+2x+3y+4z=0$.

There are more questions like this in 2.5.6 and 2.5.7 and 2.5.9.

2. True or false: a vector subspace of $\mathbb{R}^7$ defined by the simultaneous vanishing of 3 linear expressions can have dimension (a) 2, (b) 4, (c) 6.

3. True or false: Suppose $AB = I$ is an identity $n \times n$ matrix. Then
   (a) the columns of B are linearly independent;
   (b) the rows of B are linearly independent;
   (c) for every vector b there is always a solution to $Bx = b$;
   (d) for every vector b there is at most one solution to $Bx = b$;

There are more questions like this in 2.5.2, 2.5.3 and 2.5.8 on page 205.

4. Let $C = AB$ be a 4×5 matrix of rank 3, where A and B need not be square. Which of the following are possible and which could never happen?
   (a) $\operatorname{rank} B = 2$.
   (b) $\operatorname{rank} B = 4$.
   (c) The nullity of A is 2.
   (d) The nullity of B is 3.
   (e) $\operatorname{rank} C^T = 2$.

5. Which are linear maps $P^k \to P^k$?:
   (a) $T(f) = f'' + f$
   (b) $T(f) = f'' + x$
   (c) $T(f) = 0$
   (d) $T(f) = xf'' + f$
   (e) $T(f) = \int_0^x f(t)\,dt$
   (f) $T(f) = f\int_0^1 f(t)\,dt$

6. Find a basis for $\mathbb{R}^4$ which includes the vectors $(1,0,-1,0)$ and $(0,1,0,1)$. Find a basis for $\mathbb{R}^4$ that is a subset of the vectors

$$(1,2,3,4),\ (1,-1,0,1),\ (1,-4,-3,-2),\ (0,1,2,0),\ (1,-3,-1,-2).$$

7. Find the inverse of $\begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ 3 & 1 & 4 \end{bmatrix}$.

8. Find the rank and nullity of the map trace : $\mathrm{Mat}(3,3) \to \mathbb{R}$; the map $\mathrm{Mat}(3,5) \to \mathrm{Mat}(5,3)$ given by $A \mapsto A^T$; the map $\mathrm{Mat}(2,3) \to \mathrm{Mat}(2,3)$ given by $A \mapsto BA$ where B is the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

9. Find the dimension of the vector space consisting of all linear transformations $P_{\leq 2} \to P_{\leq 3}$, where $P_{\leq n}$ is the vector space of polynomials of degreem $\leq n$. Find the dimension of the vector space consisting of all linear transformations $P_{\leq 2} \to V$ where V is the subspace of $P_{\leq 3}$ consisting of polynomials with constant term zero.

Further questions: 2.11 (pages 277-282); 8, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24b, 25, 27b