Directions:
PLEASE DO NOT OPEN EXAM UNTIL DIRECTED TO DO SO.
This is a closed book exam. No books. No notes. No crib sheets. No calculators.

You are allowed 50 minutes to complete this exam.

Please show all your work on the enclosed pages. You are not allowed any scratch paper of your own.

There are 7 questions. Including this title page, there are 10 pages (the last two of which are blank).

Please make sure all the pages are here before beginning your 50 minutes of work.

Scores:
(1) (7 pts)
(2) (8 pts)
(3) (7 pts)
(4) (5 pts)
(5) (6 pts)
(6) (5 pts)
(7) (7 pts)

Total Points Possible: 45.
(1) (a) (4 points) Write a truth table for the conditional statement $p \lor (\sim q)$ using the symbols T/F for true/false, respectively.

(b) (3 points) What is the logical relationship between $p \lor (\sim q)$ and $p \Rightarrow q$? (Your answer should use at least one of the following words: equivalent, converse, contrapositive, or negation. You should prove your answer. Answers with no proof will receive no partial credit.)
Consider the following statement about real numbers \( x, y, \) and \( z \):

There exists an \( x \) such that for all \( y > x \), there exists a \( z \) for which \( z^2 + z = y \).

(a) (2 points) Write the negation of the statement above.

(b) (6 points) Determine whether the original statement is true or false. Write “true” or “false”, and then justify your answer by proving the TRUE statement – that is, either the original statement or the negation that you wrote in (a).
(3) (7 pts) If the following statement is false, explain why it is false. If the following statement is true, prove it.

The number $\log_2 7$ is irrational.
(4) (5 points). Let $A, B$ be two sets. If the following equality is true, prove it. If it is false, give a counterexample.

$$A \setminus (A \setminus B) = B \setminus (B \setminus A).$$
(5) (6 points) Let $S$ be the Cartesian coordinate plane $\mathbb{R} \times \mathbb{R}$ and define a relation $R$ on $S$ by,

$$(a, b) R (c, d) \text{ iff } a + d = b + c.$$ 

Verify that $R$ is an equivalence relation and describe the equivalence class $E_{(3,2)}$. 


(6) . (5 pts) Suppose that \( f : A \to B \) and let \( C \) be a subset of \( A \). Prove or give a counterexample:

\[
f(A) \setminus f(C) \subseteq f(A \setminus C).
\]
(7) . (7 pts) Using any result stated or proved in the text, prove the following:

If $A$ is any infinite set and $\mathbb{N}$ denotes the set of natural numbers, then the set $A \cup \mathbb{N}$ is equinumerous with $A$. (You may not quote the results of your homework here without reproving them, with the exception of the following result which you are allowed to quote without proof: Any infinite set has a countable subset.)