The complex numbers $z = a + bi$ are defined by $z = a + bi$, where $a$ and $b$ are real numbers and $i$ is the imaginary unit with $i^2 = -1$.

The complex conjugate of $z = a + bi$ is $\bar{z} = a - bi$.

Properties:
1. $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
2. $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$
3. If $z$ is a real number, then $\overline{z} = z$.

For $z = a + bi$, we define $z^* = z + bi$.

Properties:
- $z^* = \overline{z}$
- $z = z^*$ if and only if $z$ is real.

Every real number $x$ can be expressed as $x + 0i$.

We say that $z = a + bi$ is in the first quadrant if $x > 0$ and $y > 0$.

The inner product $\langle z, w \rangle$ on a complex vector space $V$ is defined as $\langle z, w \rangle = x \overline{y}$.

This is the complex version of the dot product and is valid in any $\mathbb{C}^n$.

Theorem: Let $(V, \langle \cdot, \cdot \rangle)$ be a complex inner product space. A linear map $T : V \rightarrow V$ is normal if $\langle Tz, z \rangle = \langle z, Tz \rangle$ for all $z \in V$.

If $V$ is real, we call them symmetric and skew-symmetric.

Theorem: Let $V$ be a real inner product space, and $T : V \rightarrow V$ a linear map.

If $T$ is symmetric, then $\overline{T}$ is symmetric.

If $T$ is skew-symmetric, then $\overline{T}$ is skew-symmetric.

Proof: We have $\lambda + (Tz, z)$ so $\overline{T} = (\overline{T})^*$.

If $T$ is symmetric, then $\overline{T}$ is symmetric.

If $T$ is skew-symmetric, then $\overline{T}$ is skew-symmetric.