A Taylor series centered at $a$ for the function $f(x)$ is
\[
f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n
\]

1. Fill in the following table for the function $f(x) = xe^{2x}$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$f^{(n)}(x)$</th>
<th>$f^{(n)}(0)$</th>
<th>$\frac{f^{(n)}(0)}{n!}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the table to write the third Taylor polynomial $T_3$ of $f(x) = xe^{2x}$ at $a = 0$.

2. Fill in the following table for the function $f(x) = \sqrt[3]{x}$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$f^{(n)}(x)$</th>
<th>$f^{(n)}(8)$</th>
<th>$\frac{f^{(n)}(8)}{n!}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the table to write the third Taylor polynomial $T_3$ of $f(x) = \sqrt[3]{x}$ at $a = 8$.  

3. Find the Maclaurin series using the definition for \( f(x) = \ln(1 + x) \). Start by computing \( f^{(n)}(0) \) for enough \( n \) to see a pattern.

4. Find the Maclaurin series using the definition for \( f(x) = e^{-2x} \). Start by computing \( f^{(n)}(0) \) for enough \( n \) to see a pattern.
5. Find the Taylor series using the definition for $f(x) = x^3 - 3x + 4$ at $a = 2$.

6. Find the Taylor series using the definition for $f(x) = \cos x$ at $a = \pi/2$. Start by computing $f^{(n)}(\pi/2)$ for enough $n$ to see a pattern.
A list of known Maclaurin series:

\[ \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \quad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

\[ \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \]

7. Use the list above to find a Maclaurin series for each of the following functions:

- \( f(x) = \sin(2x) \)

\[ f(x) = x e^{2x} \]

- \( f(x) = x^2 \cos(x^2) \)

\[ f(x) = e^{3x} - e^{2x} \]
Taylor’s Inequality says that if $|f^{(n+1)}(x)| \leq M$ then

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

8. Approximate $f(x) = x^{-1/2}$ by a Taylor polynomial of degree 2 at $a = 4$.

To apply Taylor’s inequality, we need a number $M$ so that $|f^{(3)}(x)| \leq M$. What is $f^{(3)}(x)$?

Assume that $3 \leq x \leq 5$. As best you can, give an upper bound for $|f^{(3)}(x)|$. This is the value $M$.

Assuming again that $3 \leq x \leq 5$, give an upper bound for $|x - 4|^3$.

Use Taylor’s inequality with $n = 2$ to give an upper bound for the error $R_2(x)$ of the approximation we found earlier.
9. Approximate \( f(x) = x \cos x \) by a Taylor polynomial of degree 4 at \( a = 0 \).

To apply Taylor’s inequality, we need a number \( M \) so that \( |f^{(5)}(x)| \leq M \). What is \( f^{(5)}(x) \)?

Assume that \(-1 \leq x \leq 1\). As best you can, give an upper bound for \( |f^{(5)}(x)| \). This is the value \( M \).

Assuming again that \(-1 \leq x \leq 1\), give an upper bound for \( |x|^5 \).

Use Taylor’s inequality with \( n = 4 \) to give an upper bound for the error \( R_4(x) \) of the approximation we found earlier.