1. Finding Values of $\tan, \cot, \sec, \csc$

2. You should be familiar with the process to find the cosine value, given the sine value and vice versa.

   In this lesson, we will define four other trig functions, the tangent, cotangent, secant and cosecant, and find their values.

3. First, using the functions sine and cosine, we will define the other four trig functions. The tangent is defined as the sine divided by the cosine. The other three functions are reciprocals. The cotangent is the reciprocal of the tangent. The cosecant, abbreviated c-s-c, is the reciprocal of the sine. And the secant is the reciprocal of the cosine. Note that each pair has one co-function. The reciprocal of sine is CO-secant, and the reciprocal of CO-sine is secant. We can find the values of these functions, based on the values of sine and cosine.

4. For example, if $\sin\theta = -\frac{3}{5}$, and $\cos\theta = \frac{4}{5}$, find the values of the other four trig functions. This is done simply by using the definitions.

5. The tangent value is the sine value divided by the cosine value, the cotangent value is the reciprocal of the tangent value, the secant value is the reciprocal of the cosine value, and the cosecant value is the reciprocal of the sine value.

6. We actually gave you more information than you needed in order to find these trig values. Once we know the sine value, the cosine value is also determined, up to a plus or a minus. Here, we use the trig version of the Pythagorean Theorem to find the cosine. Since we are given that the cosine is positive,

7. we know that $\cos\theta = +\frac{4}{5}$.

8. We could also have specified in which quadrant θ lies,

9. or we could have specified the size of θ, either as a positive angle

10. or a negative angle.

11. To recap: Given either the sine value or cosine value, the standard procedure is to use the trig version of the Pythagorean Theorem, that $\sin^2\theta + \cos^2\theta = 1$, to find the other, and then use the definitions.

12. If you are given either the secant or cosecant, first take the reciprocal to find the sine or cosine, and then follow the standard procedure.

13. Here is an example. We are given the cosecant value. We can first find the sine value by taking the reciprocal.

14. We next find the cosine value by using the the trig version of the Pythagorean Theorem.

15. Note that since the angle is between π and $\frac{3\pi}{2}$, we are in the third quadrant, and therefore we choose the negative value.

16. Now that we have the values for sine and cosine, we can find the value of tangent,

17. and then take reciprocals to find the value for cotangent

18. and secant.

19. If you are given the tangent value, use the geometric definition to find the sine and the cosine. In this example, $\tan\theta = -2$. For the time being lets ignore the negative sign and think of the tangent value as the fraction $\frac{2}{1}$, which is how we will label the opposite and adjacent sides. We will then come back and figure out where to place negative signs.

20. Since the sine value is positive, and the tangent is negative, the cosine should also be negative and the angle should be in the second quadrant.

21. When we label our second quadrant angle, the opposite side should be length 2 and the adjacent side should be length 1, and we should place a negative sign on the cosine value which corresponds to the adjacent side.

22. From there, we use the Pythagorean Theorem to find the length of the hypotenuse. And once weve drawn the triangle, we can then read off the values of sine and cosine from the geometric definition.

23. The sine value will be the opposite over the hypotenuse, and the cosine value will be the adjacent over the hypotenuse.

    From here, we can take reciprocals to find the cotangent,

24. the secant,

25. and the cosecant.

26. .

27. If you are given the cotangent value, take the reciprocal to find the tangent, and then follow the tangent procedure.

    Here we know $\cot\theta = 3$,

28. and therefore the tangent value should be the reciprocal of 3, which is $\frac{1}{3}$. We then label the appropriate triangle. The opposite over adjacent should be one over three.

29. Since the sine value is negative and the tangent value is positive, then the cosine value should also be negative, and we will be in the third quadrant.

30. We label our third quadrant triangle with opposite over adjacent $= \frac{1}{3}$, and then place a negative sign on both the 1 and the 3 since sin and cos are both negative. We then use the Pythagorean Theorem to find the length of the hypotenuse, and then read off the appropriate values.

31. The sine will be opposite over hypotenuse.

32. The cosine will be adjacent over hypotenuse.

33. And the other functions are reciprocals.

34. .

35. To recap: If you are given the cosecant or secant value, take the reciprocal to find the sine or cosine value, and then proceed as usual. If you are given the tangent value, draw a right triangle, use the Pythagorean Theorem and the geometric definitions to find sine and cosine, and then proceed as usual. If you are given the cotangent value, take the reciprocal to find the tangent value, and then follow the tangent procedure.