1. Common Functions and Their Graphs

2. You should be familiar with the Cartesian Coordinate System. In this lesson, we will discuss important facts about the graphs of many common functions.

3. Linear functions can be described in several ways, the simplest being slope-intercept form. A line is determined by its slope and where it crosses the $y$-axis.

4. Another important family of functions are power functions. The parabola $y = x^2$ goes through the origin, and the points $(1, 1)$ and $(-1, 1)$. It is symmetric with respect to the $y$-axis.

   * $y = x^3$ goes through the origin, and the points $(1, 1)$ and $(-1, -1)$. It is symmetric with respect to the origin.
   * All higher even powers of $x$, like $x^4$,
     * $x^6$ . . .
   * look similar to $y = x^2$. They all go through the origin, the points $(1, 1)$ and $(-1, 1)$, and are symmetric with respect to the $y$-axis. The higher the power, the longer the function stays close to the $x$-axis, and the steeper the graph is beyond $x = 1$.
   * All higher odd powers of $x$, like $x^5$,
     * $x^7$ . . .
   * look similar to $y = x^3$. They all go through the origin, the points $(1, 1)$ and $(-1, -1)$, and are symmetric with respect to the origin. The higher the power, the longer the function stays close to the $x$-axis, and the steeper the graph is beyond $x = 1$.

5. Exponential functions stay above the $x$-axis. That is, their range is strictly positive. They all pass through the point $(0, 1)$. They all approach the $x$-axis asymptotically to the left. The larger the base, the longer the graph stays close to the $x$-axis for negative values of $x$, and the steeper the graph climbs to the right.

6. Logarithm functions are the inverses of exponential functions. They all have a domain which is strictly positive. They all pass through the point $(1, 0)$. They all approach the $y$-axis asymptotically downward. The larger the base, the longer the bottom of the graph stays close to the $y$-axis and the shallower the graph is to the right.

7. The two wave functions, $y = \sin x$ and $y = \cos x$ are periodic with period $2\pi$.

   * They both have maximum values of 1 and minimum values of $-1$.
   * The sine wave passes through the origin, reaches its maximum value of 1 at $x = \pi/2$, returns to a $y$-value of zero at $x = \pi$, reaches its minimum value of $-1$ at $x = 3\pi/2$ and returns to a $y$-value of zero at $2\pi$. The sine wave then repeats one wave every $2\pi$.
   * The cosine wave passes through the point $(1, 0)$, drops to a $y$-value of zero at $x = \pi/2$, reaches its minimum value of $-1$ at $x = \pi$, returns to a $y$-value of zero at $3\pi/2$ and ascends to 1 at $x = 2\pi$ The cosine wave then repeats one wave every $2\pi$. 
8. The graph of $y = \tan x$ has branches separated by asymptotes. The asymptotes occur at $x = \pi/2, x = 3\pi/2\ldots$, both to the right and left of the $y$-axis. The graph passes through the origin, the point $(\pi/4, 1)$ and the point $(−\pi/4, −1)$. The tangent graph then repeats every $\pi$.

9. The inverse of the sine function, also called the arcsin, outputs angles associated with sine values. The domain is the possible sine values from -1 to 1. The range is the angles in the fourth and first quadrants, from $\pi/2$ to $\pi/2$.

10. The inverse of the cosine function, also called the arccos, outputs angles associated with cosine values. The domain is the possible cosine values from -1 to 1. The range is the angles in the first and second quadrants, from 0 to $\pi$.

11. The inverse of the tangent function, also called the arctan, outputs angles associated with tangent values. The domain is the possible tangent values which are all real numbers. The range is the angles in the fourth and first quadrants, from $\pi/2$ to $\pi/2$. 