1. (2 points) Please sketch the region of integration and change the order of integration.
\[ \int_{0}^{\pi/2} \int_{0}^{\cos x} g(x, y) \, dy \, dx \]

Answer: For any \( y \in [0, 1] \), \( x \) varies from 0 to \( \arccos y \), hence
\[ \int_{0}^{\pi/2} \int_{0}^{\cos x} g(x, y) \, dy \, dx = \int_{0}^{1} \int_{0}^{\arccos y} g(x, y) \, dx \, dy. \]

2. (4 points) Please use the polar coordinates to find the volume of the solid that is under the cone \( z = \sqrt{x^2 + y^2} \) and above the disk \( x^2 + y^2 \leq 9 \).

Answer: We have
\[ V = \iint_{D} \sqrt{x^2 + y^2} \, dA, \]
where
\[ D = \{(x, y) : x^2 + y^2 \leq 9\}. \]

In polar coordinate, we have
\[ x = r \cos \theta, \quad y = r \sin \theta, \quad dA = r \, dr \, d\theta. \]

and for points in \( D \),
\[ 0 \leq r \leq 3, \quad 0 \leq \theta \leq 2\pi. \]

Therefore
\[ V = \int_{0}^{2\pi} \int_{0}^{3} r^2 \, dr \, d\theta \]
\[ = \int_{0}^{2\pi} \left[ \frac{1}{3} r^3 \right]_{r=0}^{r=3} d\theta \]
\[ = \int_{0}^{2\pi} 9 \, d\theta \]
\[ = 18\pi. \]
3. (4 points) Please find the mass and center of mass of the lamina that occupies the region $D$ and has the given density function $\rho$, where

$$D = \{(x, y) | 1 \leq x \leq 3, 1 \leq y \leq 4\}; \quad \rho(x, y) = x.$$ 

**Answer:** The mass is the integral of the density function:

$$m = \iint_D x \, dA = \int_1^4 \int_1^3 x \, dx \, dy = \int_1^4 \left[ \frac{x^3}{3} \right]_{x=1}^{x=3} dy = \int_1^4 \frac{32}{3} \, dy = \frac{64}{3}.$$ 

We have moment about $x$-axis

$$M_x = \iint_D y \rho \, dA = \int_1^4 \int_1^3 xy \, dx \, dy = \int_1^4 \left[ \frac{x^3 y}{2} \right]_{x=1}^{x=3} dy = \int_1^4 4y \, dy = \frac{32}{3},$$

and the moment about the $y$-axis

$$M_y = \iint_D x \rho \, dA = \int_1^4 \int_1^3 x^2 \, dx \, dy = \int_1^4 \left[ \frac{x^3}{3} \right]_{x=1}^{x=3} dy = \int_1^4 \frac{26}{3} \, dy = 26.$$ 

Therefore the center of mass is

$$\left( \overline{x}, \overline{y} \right) = \left( \frac{M_y}{m}, \frac{M_x}{m} \right) = \left( \frac{26}{12}, \frac{30}{12} \right) = \left( \frac{13}{6}, \frac{5}{2} \right).$$

**Remark:** I did not require the computation of the center of mass, you just need to set up the correct formula. Please note it is **wrong** to write \( \left( \overline{x}, \overline{y} \right) = \left( \frac{M_x}{m}, \frac{M_y}{m} \right) \).