1. (5 points) Please evaluate the integral
\[ \int \int_R \frac{x - 2y}{3x - y} \, dA \]
by making an appropriate change of variables. Here \( R \) is the parallelogram enclosed by the lines \( x - 2y = 0, \ x - 2y = 4, \ 3x - y = 1, \) and \( 3x - y = 9. \)

**Answer:** Set 
\[
(1)\quad u = x - 2y, \ v = 3x - y.
\]

Then we have
\[
x = -\frac{1}{5} u + \frac{2}{5} v, \quad y = -\frac{3}{5} u + \frac{1}{5} v,
\]
and hence the Jacobian
\[
\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{vmatrix} = \begin{vmatrix}
-\frac{1}{5} & \frac{2}{5} \\
-\frac{3}{5} & \frac{1}{5}
\end{vmatrix} = \frac{1}{5}.
\]

We also know that the region \( S \) on \( uv \)-plane that corresponds to \( R \) is
\[
S = \left\{ (u, v) \in \mathbb{R}^2 : 0 \leq u \leq 4, 1 \leq v \leq 9 \right\}.
\]

Hence we have
\[
\int \int_R \frac{x - 2y}{3x - y} \, dA = \int \int_S \frac{u}{5v} \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, dA = \int \int_S \frac{u}{5v} \, dA = \int_1^9 \int_0^4 \frac{u}{5v} \, du \, dy = \int_1^9 \frac{8}{5v} \, dy = \frac{8}{5} (\ln 9 - \ln 1) = \frac{16 \ln 3}{5}.
\]
2. (5 points) Please Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where
\[
\mathbf{F}(x, y, z) = (x + y) \mathbf{i} + (y + z) \mathbf{j} + (x + z) \mathbf{k},
\]
\[
C: \mathbf{r}(t) = t \mathbf{i} + 2t \mathbf{j} + t^2 \mathbf{k}, \quad 0 \leq t \leq 1.
\]

**Answer:** Along the curve $C$, we have
\[
\mathbf{F} = (t + 2t, 2t + t^2, t + t^2) = (3t, t^2 + 2t, t^2 + t)
\]
\[
\mathbf{r}' = (1, 2, 2t).
\]
Hence
\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle 3t, t^2 + 2t, t^2 + t \rangle \cdot (1, 2, 2t) \, dt
\]
\[
= \int_0^1 (3t + 2t^2 + 4t + 2t^3 + 2t^2) \, dt
\]
\[
= \int_0^1 (2t^3 + 4t^2 + 7t) \, dt
\]
\[
= \left[ \frac{1}{2} t^4 + \frac{4}{3} t^3 + \frac{7}{2} t^2 \right]_0^1
\]
\[
= \frac{16}{3}.
\]