1. (10 pts) A loan of $5000 is to be paid off with level payments at the end of each month at 3% compounded monthly. The amount of interest paid in the 61st payment is $2.24, and the amount of principal repaid in the 42nd payment is $70.31. Find the amount of the level monthly payment and the number of months necessary to pay off the loan.

**Answer:** Let $R$ be the level monthly payment, $j$ be the effective rate per month, and $n$ be the number of months. We have

\[ Ra_n j = 5000, \]
\[ I_{61} = R(1 - v^{n-61+1}) = 2.24, \]
\[ P_{42} = Rv^{n-42+1} = 70.31, \]

where $v = 1/(1+j)$. We note that $P_{61} = Rv^{n-61+1} = P_{42}v^{-19} = P_{42}(1+j)^{19}$, hence we have

\[ R = I_{61} + P_{61} = I_{61} + P_{42}(1+j)^{19} = 2.24 + 70.31 \times (1 + 3%/12)^{19} \approx 75.97. \]

Plugging this value into (1), we have

\[ n = \log(1 - \frac{5000j}{R})/\log v \approx 72. \]

2. (10 pts) A loan is repaid with payments which start at $200 the first year and increase by $100 per year until a payment of $2000 is made, at which time payments cease. If interest rate is 5% effective, find the amount of principal in the 4th payment.

**Answer:** We denote $R_t$, $I_t$, and $P_t$ the amounts of the total payment, interest, and principal at $t$. We have $R_t = P_t + I_t$ for all $t$. In particular,

\[ P_4 = R_4 - I_4 = 500 - iB_3 \]

where $B_t$ stands for the outstanding balance after the $t$th payment, and $i = 5\%$ is the given effective rate. Using the prospective method, $B_3$ is the present value (at $t = 3$) of the 16 future payments ($500, 600, \ldots, 2000$), i.e.,

\[ B_3 = Pa_{\overline{16}|i} + Q \frac{a_{\overline{16}|i} - 16v^{16}}{i}, \]

where $P = 500$, $Q = 100$, and $v = 1/(1+i)$. From (4) and (5), we obtain $P_4 \approx -121.74$. 

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**MATH 4065 Theory of Interest, Fall 2013, Section 003**

Quiz 5 (Nov 4, 2013)

Print your name __________________________  Student ID # ___________  Score _____

* Please show your work.