Due March 15, beginning of class

Instructions: Show your work. An explicit, logical, and neat presentation of each solution is required. Numbered problems are from Meerschaert.

1. Section 5.4, Problem 10.

2. Consider the blue/fin whale problem that we discussed in class:

\[
\begin{align*}
\frac{dx}{dt} &= r_1 x \left(1 - \frac{x}{K_1}\right) - \alpha_1 xy, \\
\frac{dy}{dt} &= r_2 y \left(1 - \frac{y}{K_2}\right) - \alpha_2 xy,
\end{align*}
\]

where all constants \(r_1, r_2, K_1, K_2, \alpha_1, \alpha_2\) are positive.

(a) Consider the case:
\[
\frac{r_1}{\alpha_1} > K_2, \quad \frac{r_2}{\alpha_2} > K_1.
\]
Find all the steady states and determine their stability by examining the Jacobian matrix at each of the steady states.

(b) Consider the case:
\[
\frac{r_1}{\alpha_1} < K_2, \quad \frac{r_2}{\alpha_2} < K_1.
\]
Find all the steady states and determine their stability by examining the Jacobian matrix at each of the steady states.

3. Consider the map \(x_{n+1} = 3x_n - x_n^3\).

(a) Find all fixed points and classify their stability.

(b) Draw a cobweb starting at \(x_0 = 1.9\).

(c) Draw a cobweb starting at \(x_0 = 2.1\).

(d) Try to explain the difference between the orbits found in parts (b) and (c). For instance, can you prove that the orbit in (b) will remain bounded for all \(n\)? Or that \(|x_n| \to \infty\) in (c)?