Due April 1, beginning of class

Instructions: Show your work. An explicit, logical, and neat presentation of each solution is required.

1. Consider the equation
\[ \frac{dx}{dt} = 3x - x^3 - r. \]
The parameter \( r \) can vary over all real values. Find all the fixed points and identify the value(s) or \( r \) at which bifurcations take place. State what kind of bifurcation you find, and draw the bifurcation diagram (\( x^* \) against \( r \)).

2. For each equation below, find the values of \( r \) at which the bifurcations occur and classify each bifurcation as saddle-node, transcritical, or pitchfork. Then, sketch the bifurcation diagram of the fixed points \( x^* \) vs. \( r \).

(a) \[ \frac{dx}{dt} = rx - \frac{x}{1 + x} \]
(b) \[ \frac{dx}{dt} = rx - \frac{x}{1 + x^2} \]
(c) \[ \frac{dx}{dt} = 5 - re^{-x^2} \]