1. Problems from textbook:
   (a) Section 1.4: 2, 3, 4.
   (b) Section 1.5: 5, 6, 11.

2. For a $m \times n$ matrix $A$ denote by $\sigma_1(A) \geq \sigma_2(A) \geq \cdots \sigma_r(A) > 0$ the singular values of $A$.

   (a) Let $T_p$ be the set of all $m \times n$ matrices of rank $p$. Show that
      $$\min_{B \in T_p} \sigma_1(A - B) \geq \sigma_{p+1}(A)$$
      if $p + 1 \leq r$. (Hint: We did this proof in class, so you can just write that down here.)

   (b) Show that
      $$\min_{B \in T_p} \sigma_k(A - B) \geq \sigma_{p+k}(A)$$
      if $p + k \leq r$. (Hint: Mimic the argument of previous problem.)

   (c) Given a singular value decomposition $A = U \Sigma V^*$ where $V = (v_1, \cdots, v_r)$, $U = (u_1, \cdots, u_r)$. Let
      $$A_p = \sum_{k=1}^{p} \sigma_k(A) u_k v_k^*, \quad p \leq r - 1.$$ 
      Show that, for $p \leq r - 1$,
      $$\min_{B \in T_p} \|A - B\|_F \geq \|A - A_p\|_F = \sqrt{\sum_{k=p+1}^{r} (\sigma_k(A))^2},$$
      where $\|\cdot\|_F$ is the Frobenius norm. This shows that $A_p$ is the best $p$-rank approximation of $A$ in the Frobenius norm.