1. Problems from Notes: Section 12: 1, 2, 5, 7.

2. 1.21 in Holmes.

3. Consider the following eigenvalue problem:

\[ \frac{\partial^2 u}{\partial x^2} + \epsilon f(x)u = \lambda u, \quad \text{in } \mathbb{R}/2\pi\mathbb{Z}, \]

(1)

where \( f \) is a smooth periodic function. Here, \( \lambda \) is an eigenvalue and \( u \) is the eigenfunction.

(a) Find all eigenvalues and eigenfunctions of the above problem when \( \epsilon = 0 \).

(b) Discuss what happens with the eigenvalues and eigenfunctions when \( \epsilon \neq 0 \) but small.

4. Consider the eigenvalue perturbation problem for the matrix \( A + \epsilon B \) where \( \epsilon > 0 \) is a small parameter. Let \( \lambda_0 \) be the eigenvalue from which we perform perturbations. Suppose the eigenvalue \( \lambda_0 \) is an eigenvalue of geometric multiplicity 1 but of algebraic multiplicity 2 (that is to say, eigenspace is one-dimensional but the generalized eigenspace is 2-dimensional). Find the leading order correction to the eigenvalue and eigenvector.