1. The following problem is mostly a review of what we did in class. Consider
\[ y'' + \epsilon f(y, y') + y = 0, \quad t > 0 \tag{1} \]
where \( \epsilon \) is small and \( f \) is some smooth function.

(a) Use the multiple time scale expansion:
\[ y = y_0(t_1, t_2) + \epsilon y_1(t_1, t_2) + \cdots, \quad t_1 = t, \ t_2 = \epsilon t \tag{2} \]
and show that \( y_0 \) can be written as:
\[ y_0 = A(t_2) \cos(\tau), \quad \tau = t_1 + \phi(t_2) \tag{3} \]
where \( \phi(t_2) \) is some function of \( t_2 \).

(b) Show that the \( O(\epsilon) \) equation is:
\[ \frac{\partial^2 y_1}{\partial t_1^2} + y_1 = 2 \left( \frac{\partial A}{\partial t_2} \sin(\tau) + A \frac{\partial \phi}{\partial t_2} \cos(\tau) \right) - f \left( y_0, \frac{\partial y_0}{\partial \tau} \right) \tag{4} \]

(c) In order to avoid secular terms, show that the conditions to be satisfied are:
\[ \frac{\partial A}{\partial t_2} = \frac{1}{2\pi} \int_{0}^{2\pi} f \left( y_0, \frac{\partial y_0}{\partial \tau} \right) \sin(\tau) d\tau \tag{5} \]
\[ A \frac{\partial \phi}{\partial t_2} = \frac{1}{2\pi} \int_{0}^{2\pi} f \left( y_0, \frac{\partial y_0}{\partial \tau} \right) \cos(\tau) d\tau \tag{6} \]

(d) Use the above result to discuss the steady state and limit cycle when \( f = (y^4 - 1)y' \).

2. Problem 3.15 from Holmes.

3. Problem 3.35 from Holmes.