1. Suppose

\[ f(x) = \frac{x}{x^2 + 1}, \quad f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}, \quad f''(x) = \frac{2x(x^2 - 3)}{(x^2 + 1)^3} \]

Answer the following questions.

1. What is the domain for \( f \)?

2. Is \( f \) is an even or an odd function, and whether the curve \( y = f(x) \) is symmetric with respect to the \( y \)-axis or the origin?

3. Find all the intercepts of \( f \).

4. Find all vertical, and horizontal, asymptotes, of \( f(x) \) (if there are any.)

5. Find all the critical numbers of \( f(x) \) (if there are any.) Identify the intervals over which \( f(x) \) is increasing / decreasing. Find all the local maximum and local minimum.

6. Identify the intervals over which \( f(x) \) is concave up / concave down. Find all the inflection points of \( f \).

7. Sketch the curve of \( y = f(x) \).
2. Evaluate the following limit

\[ \lim_{x \to 1} \frac{x}{x-1} - \frac{1}{\ln x} \]

Hint: find the common denominator of \( \frac{x}{x-1} \), and \( \frac{1}{\ln x} \), write the two fractions as one fraction. Then apply L’Hospital’s rule.

3. Find the derivative of the function

\[ f(x) = (1 - 2x)^{1/x} \]

Hint: use log differentiation in Sec 3.6. Caution: this is NOT a limit problem. It is different from the question evaluating \( \lim_{x \to 0} (1 - 2x)^{1/x} \) in Sec 4.4, which needs L’Hospital’s rule.
4. Let

\[ f(x) = \frac{1}{(x + 1)^2} \]

On which one of the following intervals (a): \([-2, 0]\); (b): \([0, 2]\), that one can apply the mean Value Theorem to \(f(x)\). Why? If we can apply MVT to one of these two intervals (or both), then find all numbers that satisfy the conclusion of the Mean Value Theorem.

5. Find all the critical numbers of

\[ f(x) = 4x^{\frac{2}{5}} - x^{\frac{8}{5}} \]
6. A cylindrical tank with diameter 10 m is being filled with water at a rate of 2 m$^3$/min. How fast is the height of the water increasing?

7. Use implicit differentiation to find an equation of the tangent line to the curve

\[ x^2 + y^2 = (2x^2 + 2y^2 - x)^2 \]

at the point \( (0, \frac{1}{2}) \).
Derivatives formulas:
- \((c)' = 0; (ax + b)' = a; (x^n)' = nx^{n-1}\)
- \((\sin x)' = \cos x; (\cos x)' = -\sin x; (\tan x)' = \sec^2 x; (\sec x)' = \tan x \cdot \sec x\)
- \((\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}; (\tan^{-1} x)' = \frac{1}{1+x^2}; (\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}\)
- \((e^x)' = e^x; (ax)' = a \cdot x \cdot \ln a; (\ln x)' = \frac{1}{x} ; (\log_a x)' = \frac{1}{x \ln a}\)
- \((cf)' = cf'; (fg)' = f'g + fg'; \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}; (f(g))' = f'(g) \cdot g'\)
- \(y = y(x),\) then \((\ln y)' = \frac{y'}{y}; (y^2)' = 2y \cdot y', \ldots,\)

Derivative theorems:
- Linearization of \(f\) at \(a\): \(L(x) = f(a) + f'(a)(x - a),\) and \(L(x) \approx f(x)\)
- MVT: If \(f\) is continuous on \([a,b]\) and differentiable on \((a,b)\) then there exists \(c\) in \((a,b)\) that satisfies \(f'(c) = \frac{f(b) - f(a)}{b-a}\)
- Critical numbers: \(f' = 0\) or \(f'\) D.N.E.; \(f' > 0:\) increasing; \(f' < 0:\) decreasing;
- Inflection points: \(f'' = 0\) and concavity changes; \(f'' > 0:\) concave up; \(f'' < 0:\) concave down

Limits formulas:
- \(\frac{1}{\pm\infty} = 0; \frac{1}{0^+} = \pm\infty; e^{\pm\infty} = \infty; e^{\pm\infty} = 0; \ln \infty = \infty; \ln 0^+ = -\infty;\)
- \(\tan^{-1}(\pm\infty) = \pm\frac{\pi}{2}; a > 1: a^{\infty} = \infty; a^{-\infty} = 0;\)
- \(\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{x}{\sin x} = 1; \text{ L'H: } \lim_{x \to a} \frac{f}{g} = \lim_{x \to a} \frac{f'}{g'}\)

Algebraic/Geometric/Trigonometric formulas:
- \(a^2 - b^2 = (a-b)(a+b); a^3 - b^3 = (a-b)(a^2 + ab + b^2)\)
- \((x^a)^b = x^{ab}; \ln(xy) = \ln x + \ln y; \ln \frac{x}{y} = \ln x - \ln y; \ln x^a = a \ln x;\)
- \(\ln e^x = x; e^{\ln x} = x\)
- Area of Circle: \(\pi r^2;\) Circumference of Circle: \(2\pi r;\)
- Area of Triangle: \(\frac{1}{2} \cdot \text{base} \cdot \text{height}\)
- Volume of Sphere: \(\frac{4}{3}\pi r^3;\) Surface Area of Sphere: \(4\pi r^2;\)
- Volume of Cylinder: \((\text{height}) \times (\text{area of base});\)
- Volume of Cone: \(\frac{1}{3}(\text{height}) \times (\text{area of base})\)
- \(\sin^2 x + \cos^2 x = 1; \sin(2x) = 2 \sin x \cos x; \cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x\)