MATH 4567, Fall 2023
HOMEWORK PROBLEMS No. 3
Due on November 15

Problem 1. Solve the temperature problem:

$$\begin{aligned} &u_t \;=\; k\,u_{xx}, && u = u(x,t),\;\; 0 < x < 1,\;\; t > 0\;\; (k > 0 \text{ parameter}) \\ &u_x(0,t) = u_x(1,t) = 0, && \\ &u(x,0) = \tfrac{1}{4}\,x^2. && \end{aligned}$$

Your answer will have the form of an infinite functional series.

Problem 2. *a*) Solve the boundary value problem:

$$\begin{aligned} &u_{tt} = 4\,u_{xx}, && u = u(x,t),\;\; 0 < x < 1,\;\; t > 0 \\ &u(0,t) = u(1,t) = 0, && \\ &u_t(x,0) = 0, && \\ &u(x,0) = x(1-x). && \end{aligned}$$

Your answer will have the form of an infinite functional series.
b) Write down separately the first two terms of that functional series.

Problem 3. Solve directly for the eigenvalues and normalized eigenfunctions:

a) No. 1 on page 225;
b) No. 2 on page 225;
c) No. 3 on page 225.

Problem 4. *a*) Given parameters $c > 0$ and $\beta > 0$, show that the Sturm-Liouville boundary value problem

$$\begin{aligned} &y'' + \lambda y = 0, && y = y(x),\;\; 0 \le x \le c, \\ &y'(0) = \beta\, y(0), && \\ &y'(c) = \beta\, y(c), && \end{aligned}$$

has exactly one negative eigenvalue λ_0 and that this eigenvalue is independent on $c > 0$. Find λ_0 and an associated eigenfunction $y_0(x)$.

b) Determine whether or not $\lambda = 0$ is an eigenvalue. If yes, find an associated eigenfunction.
c) Describe all positive eigenvalues and associated eigenfunctions.