COURSE ANNOUNCEMENT MATH 8590: TOPICS IN PDE

In this topics course, we will study problems in nonlinear partial differential equations inspired by dynamics of incompressible fluid flows. The main focus is to introduce tools to understand asymptotic stability of coherent structures of the two dimensional Euler equation. These coherent structures include vortices and shear flows. The study of vortices and shear flows is a classic topic in hydrodynamics, going back to pioneers such as Kelvin, Helmholtz, Orr, Rayleigh, among many others. Recently there has been tremendous progress on asymptotic stability of vortices and shear flows at both linear and nonlinear level. The topics course will give a detailed presentation of many of these latest development.

A fundamental relaxation mechanism in two dimensional Euler equation is inviscid damping through mixing of vorticity field, which leads to decay of the associated velocity field. Numerical simulations of two dimensional flows at high Reynolds [1] show that coherent structures such as vortices form naturally and may become the dominant feature over long periods of time. Rigorously establishing the numerically observed phenomena is mathematically challenging and requires deep and sophisticated analysis.

Some of the tools that have been used to understand the asymptotic stability of coherent structures for $2d$ Euler include: spectral analysis and resolvent estimates of the linearized flow which is not self adjoint and involves continuous spectrum; precise description of singularities of generalized eigenfunctions using Fourier transforms; delicate energy functionals designed to control resonances in the nonlinearity. The combination of these tools has led to the proof of asymptotic stability of shear flows close to Couette flows and that of point vortices. The case of general monotone shear flows also seem to be within reach now. However, the important case of general vortices is still open and seems to require significant new ideas.

One of the important goals of the course is to introduce various analytic tools, including spectral analysis, Fourier analysis, as well as perturbation theory, that are essential in studying asymptotic stability in fluid dynamics. These tools have a much wider range of applicability than the setting we consider. Another goal is to bring students interested in fluid dynamics to the forefront of an exciting area of research which still has many deep open problems.

The schedule of the class may be adjusted to suit students' need. The final grade will be based on homework assignments.

Reference papers (more references will be introduced as the class progresses):

- J. Bedrossian and N. Masmoudi, *Inviscid damping and the asymptotic stability of planar shear fows in the 2D Euler equations, Publ. Math. Inst. Hautes Etudes Sci.*, 122 (2015), 195-300.
- L. Faddeev, *On the theory of the stability of plane-parallel fows of an ideal fluid*, Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova Akademii Nauk SSSR, Vol. 21, pp. 164-172, 1971
- J. McWilliams, *The emergence of isolated coherent vortices in turbulent flow*, J. Fluid Mech. 146 (1984), 21-43

[1] This is some times called "free decaying two dimensional turbulence". There are many interesting videos of numerical simulations on YouTube.