Math and Climate Seminar IMA

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Mathematics and Climate Research Network

Joint MCRN/IMA Math and Climate Seminar

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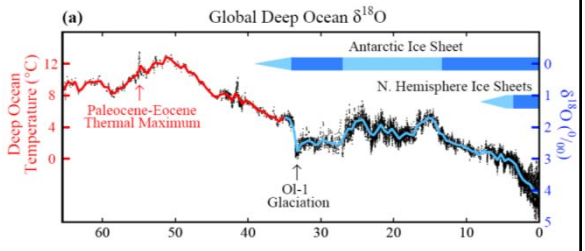
Recent Developments in the Theory of Glacial Cycles

Richard McGehee

Seminar on the Mathematics of Climate
IMA, MCRN, School of Mathematics
October 2, 2012

Glacial Cycles

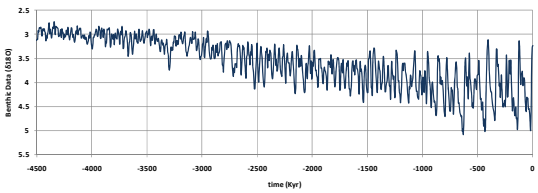
Temperatures in the Cenozoic Era



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

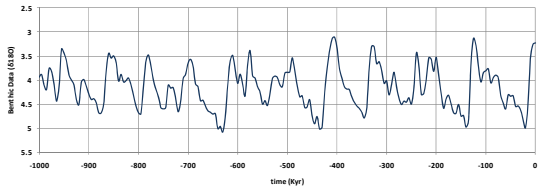
^{18}O in Foraminifera Fossils During the Past 4.5 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr

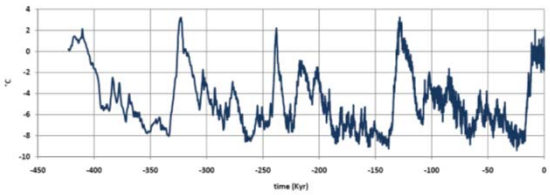


Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

Glacial Cycles

What Causes Glacial Cycles?

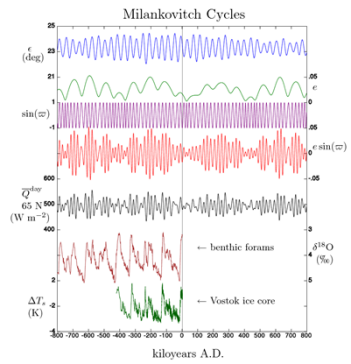
Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

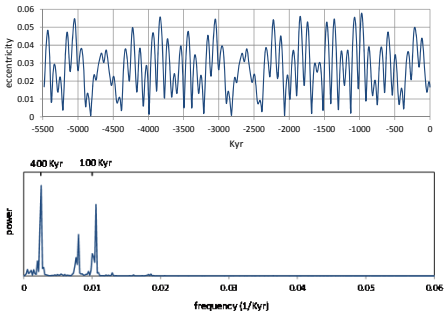
Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

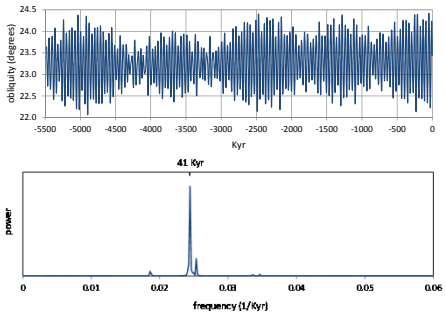
Glacial Cycles

Eccentricity



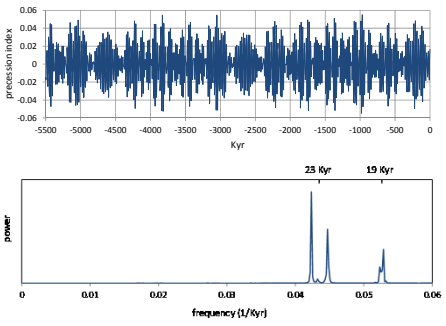
Glacial Cycles

Obliquity



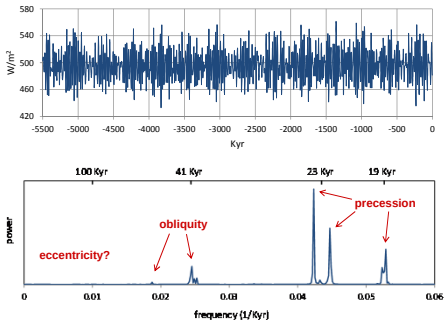
Glacial Cycles

Precession Index



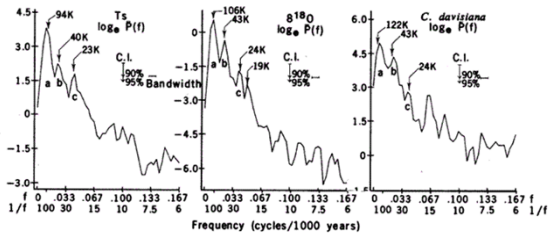
Glacial Cycles

Summer Solstice 65°N



Glacial Cycles

Climate Response, Hays, et al

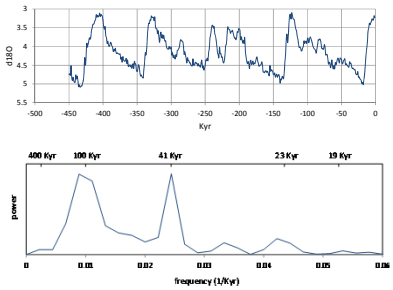


Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Hays with Modern Data



Glacial Cycles

Hays Summary

Increasing contribution	Forcing	Response
↑	precession	eccentricity
↑	obliquity	obliquity
↑	eccentricity	precession

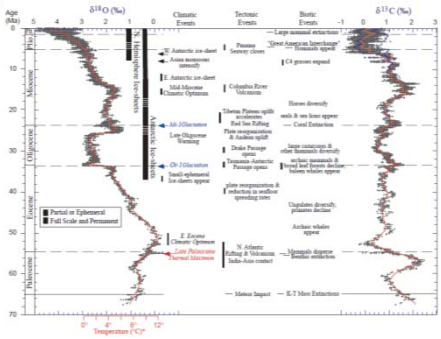
The explanation is that there are nonlinear feedbacks.

Are there other explanations?

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

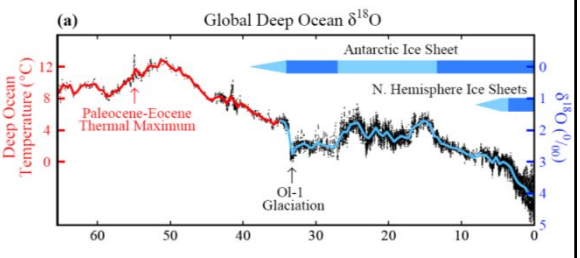
Cenozoic Era



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Temperatures in the Cenozoic Era

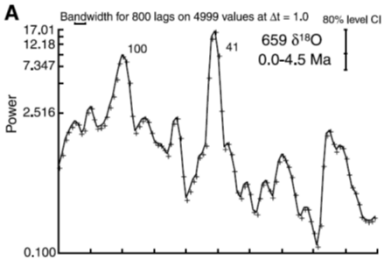


Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

Climate Response (Zachos, et al)

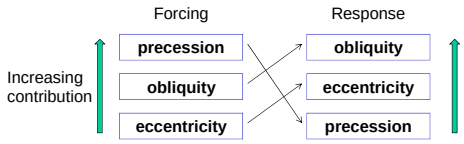
Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Zachos Summary



Nonlinear effects?

Other explanations?

Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Why such a small precession contribution?

Incoming **So**lar Radi**ation** (*Insolation*), averaged over the entire globe and over a full year, depends only on eccentricity e, not on either obliquity or precession.

$$Q(e)=\frac{Q_0}{\sqrt{1-e^2}}$$

Insolation as a function of latitude, averaged over a full year, depends on eccentricity e and obliquity β, but not precession.

$$I=Q(e)s(y,\beta)$$

where

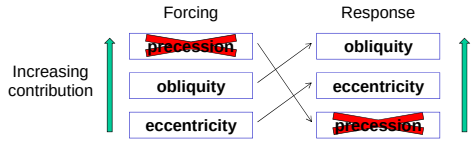
$$s(\varphi,\beta)=\frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-(\cos\varphi\sin\beta\cos\theta-\sin\varphi\cos\beta)^2}\cos\theta d\theta$$

$\varphi=$ latitude

Glacial Cycles

Zachos Summary (Revised)

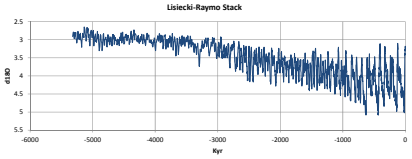
If we assume that glaciation depends on annual average insolation instead of insolation at summer solstice, then forcing and response are aligned.



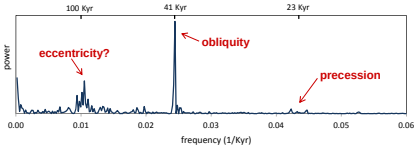
Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 5.3 Myr

data



power spectrum



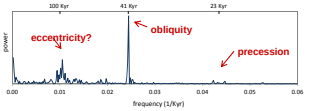
Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,**20**, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

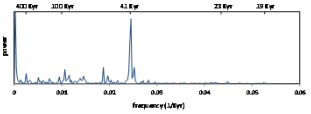
Last Million Years is Different

A transition occurred about one million years ago: the amplitude increased and the dominant period changed from 41 kyr to 100 kyr.

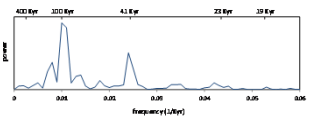
-5 to 0 Myr



-5 to -1 Myr



-1 to 0 Myr



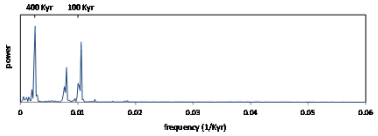
Glacial Cycles

What's up with the Last Million Years?

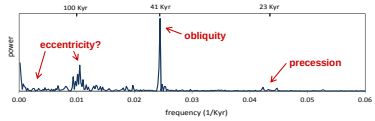
100,000 Year Problem: Why did the eccentricity signal become so dominant during the last million years?

400,000 Year Problem: If the last million years is dominated by eccentricity, what happened to the 400,000 year cycle?

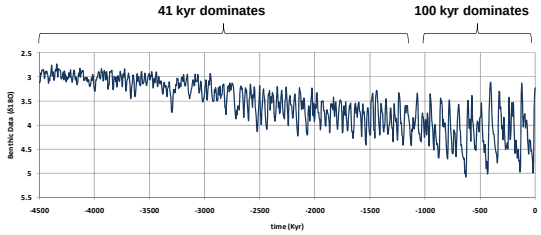
eccentricity



data



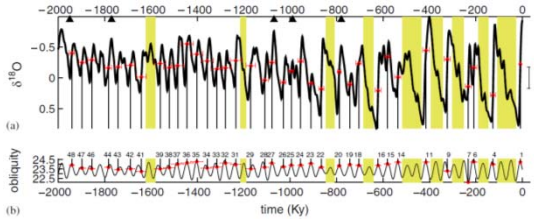
Glacial Cycles



What's up with the last million years?
Did eccentricity reassert itself? Or something else?

Glacial Cycles

Huybers' Analysis of Deglaciations



Red dots: deglaciations.

Peter Huybers, "Glacial variability over the last two million years: an extended depth-derived agemodel, continuous obliquity pacing, and the Pleistocene progression," *Quaternary Science Reviews* **26**, 37-55 (2007).

Glacial Cycles

Huybers' Analysis of Deglaciations

$$V_t = \begin{cases} V_{t-1} + \eta & \text{if } V_t < T_t \\ 0 & \text{if } V_t \geq T_t \end{cases}$$

$$T_t = at + b - c\theta_t'$$

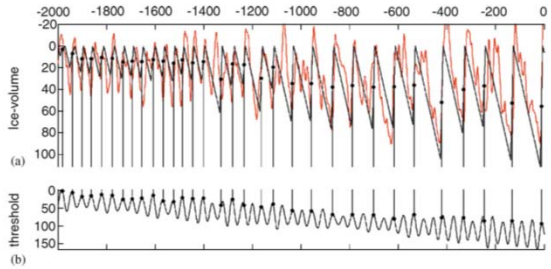
V_t : ice volume at time t
T_t : threshold variable
η : rate of increase of ice volume
θ_t' : normalized obliquity

Units and constants

t : Kyr
V : chosen so that $\eta = 1$.
θ': mean zero and variance one
$a = 0.05$
$b = 126$
$c = 20$

Glacial Cycles

Huybers' Analysis of Deglaciations



Glacial Cycles

Huybers' Analysis of Deglaciations

The deglaciations are triggered by obliquity cycles, but sometimes they don't trigger. When cycles are skipped, the deglaciations can be separated by 80 Kyr or 120 Kyr, creating the appearance of 100 Kyr cycles.

Glacial Cycles

Huybers' Analysis of Deglaciations

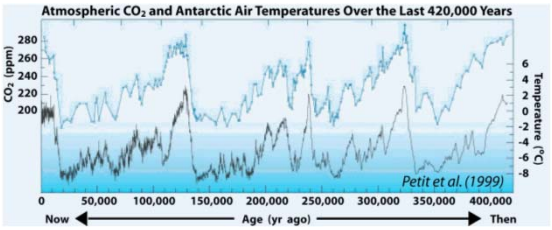
Huybers' model produces the decline in temperature and the increase in period and amplitude of the glacial cycles, but it depends heavily on an unspecified decline in the sensitivity of the triggering mechanism over last two million years.

What about greenhouse gases and the carbon cycle?

Andrew Hogg suggested a model incorporating the carbon cycle.

Glacial Cycles

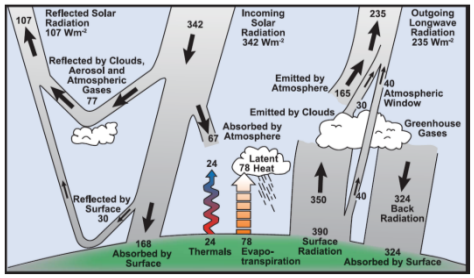
CO_2 as Feedback



Pam Martin, University of Chicago, 2010

Glacial Cycles

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
`http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf`

Glacial Cycles

Hogg's Model

surface temperature

$$c\frac{dT}{dt} = S(t) + G(C) - \sigma T^4,$$

atmospheric carbon

$$\frac{dC}{dt} = V - (W_0 + W_1 C) + \beta (C_{\max} - C)\max\left(\frac{dT}{dt} - \varepsilon, 0\right).$$

weathering; CO2 outgassing; volcanos

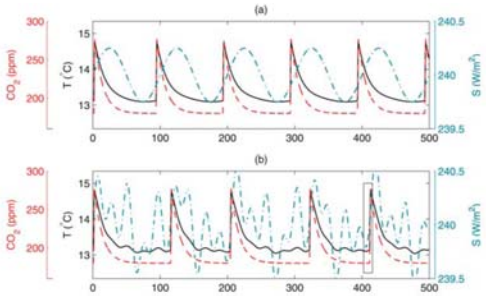
$$S(t) = \bar{S} + \sum_i S_i \sin\left(\frac{2\pi t}{\Gamma_i}\right)$$ insolation

$$G(C) = \bar{G} + A\ln\left(\frac{C}{C_0}\right)$$ greenhouse forcing

Andrew McC. Hogg, "Glacial cycles and carbon dioxide: A conceptual model," *Geophysical Research Letters* **35** (2008).

Glacial Cycles

Hogg's Model



Glacial Cycles

Hogg's Model

Hogg's model shows how the carbon cycle can act as a feedback amplifying and modifying the insolation forcing, but the forcing is somewhat artificial, and the triggering mechanism is difficult to justify.

What if the 100,000 year glacial cycle is not driven by eccentricity, but is a natural oscillation of the Earth's climate?

Saltzman and Maasch suggested just such a model.

Glacial Cycles

Salzman-Maasch Model

Milankovitch forcing

global ice mass $\longrightarrow$ $\dot{X} = -X - Y - uM(t)$

atmospheric CO_2 $\longrightarrow$ $\dot{Y} = -pZ + rY + sZ^2 - Z^2 Y$

deep ocean temperature $\longrightarrow$ $\dot{Z} = -q(X + Z)$

Barry Salzman and Kirk A. Maasch, "A Low-Order Dynamical Model of Global Climatic Variability Over the Full Pleistocene," *Journal of Geophysical Research* **95** (D2), 1955-1963 (1990)

Glacial Cycles

Salzman-Maasch Model

unforced

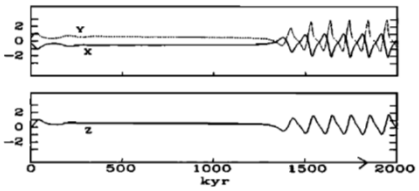


Fig. 7. Unforced (free) solution for (4)–(6) with $q = 1.2$, $s = 0.8$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

forced

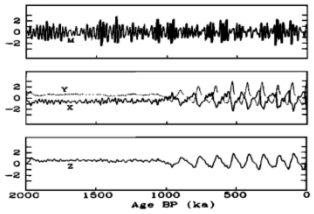


Fig. 8. Forced solution for (4)–(6) with $q = 1.2$, $s = 0.8$, $u = 0.7$, and p and r varying linearly between $0.8 \rightarrow 1.0$ and $0.7 \rightarrow 0.8$, respectively. (Top) Normalized 65°N July insolation curve (M) used as forcing. (Middle) Model solutions for X (solid) and Y (dashed). (Bottom) Model solution for Z.

Glacial Cycles

Salzman-Maasch Model

The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

Seems like a nice idea, but it is not widely accepted as the explanation.

Glacial Cycles

Questions

1. Did eccentricity play any role during the last million years? Is the apparent 100 kyr cycle an artifact (Huybers)? Is it an intrinsic cycle in the climate system that coincidentally has a period of 100,000 years (Maasch and Saltzman)?
2. Is the CO_2 feedback sufficient to explain the increasing amplitude and period of the glacial cycles during the last million years, *i.e.*, is it the mechanism behind the Huybers model.
3. Where does the atmospheric CO_2 go during the glacial maxima? The ocean? The land?
4. What will be the effect of the anthropogenic CO_2?