Math 3592H Honors Math I
Quiz 1, Thursday Sept. 22, 2016

Instructions:
15 minutes, closed book and notes, no electronic devices.
There are two problems, worth a total of 20 points.

1. (8 points total; 2 points each part)
Let A, B, C be matrices that represent linear transformations T_A, T_B, T_C (so $A = [T_A], B = [T_B], C = [T_C]$ in our book's notation), where

$$\begin{aligned} T_A &: \mathbb{R}^2 \to \mathbb{R}^5, \\ T_B &: \mathbb{R}^5 \to \mathbb{R}^3, \\ T_C &: \mathbb{R}^3 \to \mathbb{R}^2. \end{aligned}$$

What are the dimensions of these matrices?

(i) A

5x2

(ii) $\overset{3\times5}{B}\overset{5\times2}{A}$

3x2

(iii) $(CB)^\top$

$\underset{2\times3}{C}\underset{3\times5}{B}$ is 2×5, so $(CB)^\top$ is 5×2

(iv) $\underset{5\times2}{A}\underset{2\times3}{C}\underset{3\times5}{B}\underset{5\times2}{A}$

5x2

2. (12 points total; 4 points each part)
Which of these maps $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation? If it is

- linear, write down the matrix $A = [T]$ such that $T(\overline{v}) = A\overline{v}$,
- not linear, explain why not.

(i)

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5y - 99x \\ 6x - y \end{bmatrix} . = \begin{bmatrix} -99 & 5 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

<u>linear</u>: $A = \begin{bmatrix} -99 & 5 \\ 6 & -1 \end{bmatrix} = \left[T\begin{bmatrix} 1 \\ 0 \end{bmatrix}, T\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$

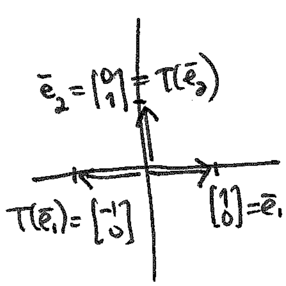
(ii)

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2 \\ y - 3 \end{bmatrix} . = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

<u>Not</u> linear, e.g. $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$T\left(2\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \neq 2\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(iii) T = reflection in $\mathbb{R}^2$ through the y-axis as a line of symmetry.



<u>linear</u>:

$$A = \left[T\begin{bmatrix} 1 \\ 0 \end{bmatrix}, T\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$