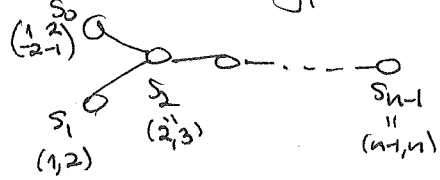
CPS 10/24/14 V. Reiner

Ref: Roichman-R, "Diameter of reduced words and galleries"

Q: In the graph of $V = \{$ reduced words for the longest element w_0 in type $D_n \}$ $\underline{a} = (s_{a_1}, s_{a_2}, \ldots, s_{a_{n^2-n}})$

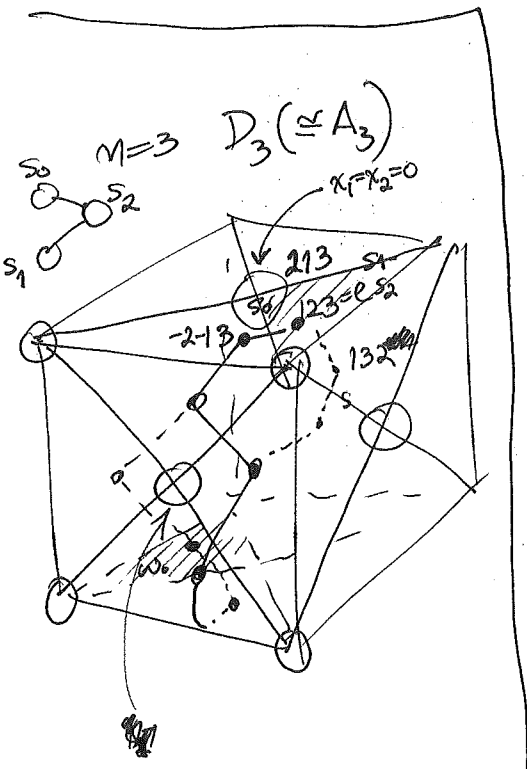


with $E = \{$ braid moves $s_i s_j = s_j s_i$ if $\{s_i, s_j\}$ not an edge here
$s_i s_j s_i = s_j s_i s_j$ if $\{s_i, s_j\}$ is an edge $\}$

does the diameter equal or exceed the "obvious" lower bound ←

= # of codim 2 intersection subspaces in the type D_n reflection hyperplane (codim 1) arrangement

(valid for all arrangements)



$$= \frac{4}{2}\binom{n}{2, 2, n-4} + 4\binom{n}{3} + \binom{n}{2}$$

$\{x_i = \pm x_j\} \cap \{x_k = \pm x_l\}$ i,j,k,l all different

$\{x_i = \pm x_j \pm x_k\}$ i,j,k different

$\{x_i = x_j = 0\}$ $(= \{x_i = x_j\} \cap \{x_i = x_j\})$ i,j different

$$= \frac{1}{6} n(n-1)(3n^2 - 11n + 13) \quad \overset{n=3}{\rightsquigarrow} \quad \frac{1}{6}(3)(2)\left(\underbrace{3 \cdot 3^2}_{27} - 11 \cdot 3 + 13\right) = 7$$

Equality in types A, B via supersolvability and in dim ≤ 3 via Cordovil and in type D_4 ○—○ via computer.