C.P.S. 10/3/14 P. Pylyavskyy

Infinite Toeplitz matrices

$$\begin{bmatrix} \ddots & 1 & a_1 & a_2 & & \\ & & 1 & a_1 & a_2 & \\ & & & 1 & a_1 & \\ & 0 & & & 1 & \ddots \end{bmatrix} \iff \text{power series } 1+a_1t+a_2t^2+\dots = a(t)$$

The ~~groups~~ under multiplication correspond.

Consider the ~~subset of~~ group (TP) ~~totally~~ positive matrices,

i.e. all minors ≥ 0

$a_1, a_2, \dots \geq 0$

$a_1^2-a_2,\ a_2^2-a_4,\ a_3^2-a_2a_4, \dots \geq 0$

THM (Edrei-Thoma)

TP matrices $a(t)$ can be written

$$a(t) = e^{\gamma t}\frac{\prod_i (1+\alpha_i t)}{\prod_j (1-\beta_j t)} \quad \text{where } \gamma, \alpha_i, \beta_j \geq 0$$

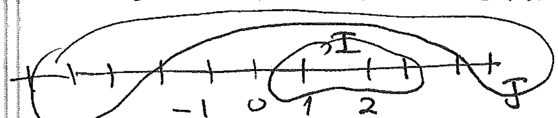
$\alpha_1 \geq \alpha_2 \geq \dots$

$\beta_1 \geq \beta_2 \geq \dots$

Now consider infinite symmetric Toeplitz matrix

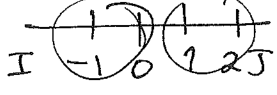
$$\begin{array}{c} \quad\ \ -1 \qquad\ 0 \qquad\ 1 \qquad\ 2 \\ \begin{bmatrix} \ddots A & -a_1 & -a_2 & -a_3 \\ -a_1 & A & -a_1 & -a_2 \\ -a_2 & -a_1 & A & -a_1 \\ & & & \ddots \end{bmatrix} \end{array} \qquad A = \sum_i a_i < \infty$$

with the condition that circular minors $\Delta_{I,J}$ are ≤ 0



e.g. $-a_1 \leq 0$

$$\Delta_{I,J} = \begin{vmatrix} -a_2 & -a_3 \\ -a_1 & -a_2 \end{vmatrix} = a_2^2 - a_1a_3 \leq 0$$

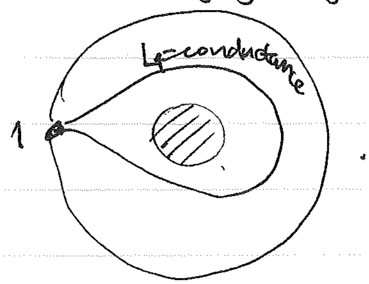


PROBLEM: Find an Edrei–Thoma–style characterization of these matrices.

Motivation: Elec. network response matrices

boundary vertices: 1, 2, ...

Put 1V battery at i and measure current through j =: (i,j) entry of response matrix



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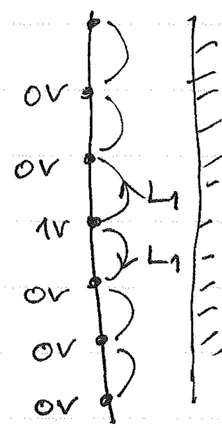
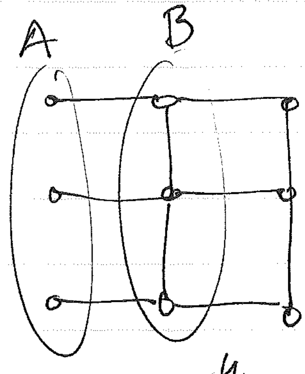


$$M = \begin{bmatrix} \ddots & 2L_1 & -L_1 & & 0 \\ & -L_1 & 2L_1 & -L_1 & \\ & 0 & -L_1 & 2L_1 & -L_1 \\ & & & -L_1 & 2L_1 \; \ddots \end{bmatrix}$$

Building blocks ~~construct~~ new ones via

Schur complement of X in $\begin{bmatrix} X & Y \\ Z & T \end{bmatrix}$ is $K = X - YT^{-1}Z$

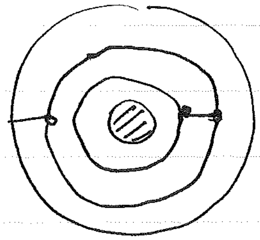


$$\begin{array}{c|cc} & A & B \\ \hline A & X & Y \\ B & Z & T \end{array}$$

get rid of B $\rightsquigarrow$ as boundary vertices

$$\begin{array}{cc} & A \\ A & \begin{bmatrix} K \\ \shortparallel \\ X - YT^{-1}Z \end{bmatrix} \end{array}$$

Can one answer the problem by making some basic building blocks, & building via Schur complements



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