C.P.S. 9/12/2014
P. Pylyavskyy

Consider graphs which are 1) bipartite
                                2) internal vertices of deg 3 } tensor diagrams
   or webs
                                    3) planar
                                                          for SL_3

Take n points • around a circle
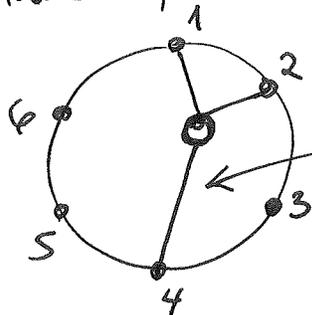
Tensor diagrams $\longleftrightarrow$ elements of coord ring
   of $Gr(3,n)$

max minor $\triangle_{124}$
in cols 1,2,4

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6=n \end{matrix}$$
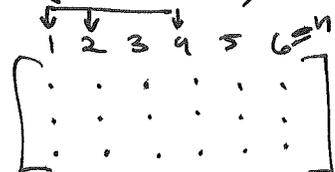
using this relation
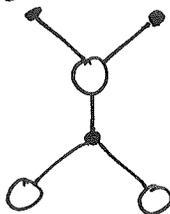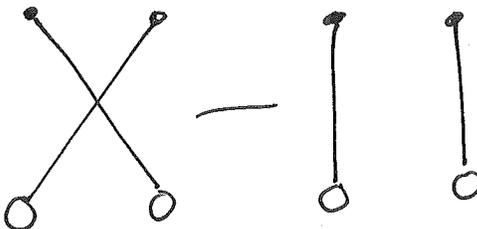
repeatedly.

e.g.
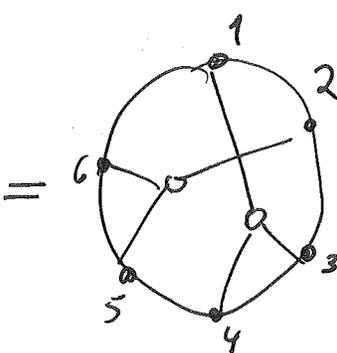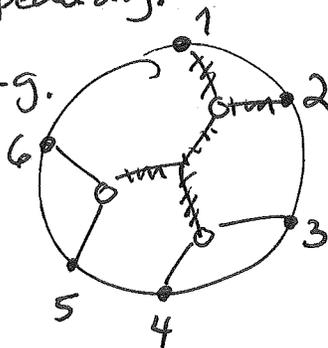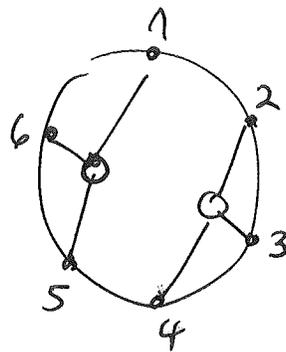
$$= \triangle_{134}\triangle_{256} - \triangle_{156}\triangle_{234}$$

More relations:
that let one
get rid of
short cycles

$$\left[\bigcirc\right] = 3$$

$$\left[-\bigcirc-\right] = -2[\ -\ ]$$

} Kuperberg

$$\left[\bowtie\right] = [\ |\ |\ ] + [\ =\ ]$$

Cluster algebras
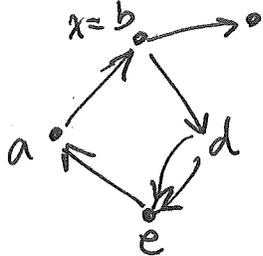
click on x

$\rightsquigarrow$ $bb' = cd + a$

define x' via $\quad b' = \dfrac{cd+a}{b}$

$$x'x = \prod_{x \to y} y + \prod_{x \leftarrow y} y$$

J. Scott's thesis tells one how to set up such a picture and label the Δ_{ijk} on vertices.

<u>CONJ</u>: All cluster variables gen'd by clicking are webs.

Here is a cluster in $Gr(3,6)$:

e.g.

$\rightsquigarrow$ click on Δ_{235}

$\rightsquigarrow$ click on Δ_{135}

click on Δ_{235}

$\rightsquigarrow$ $\Delta_{235}\underbrace{\cancel{\Delta}}_{4} = \Delta_{123}\Delta_{345} + \Delta_{135}\Delta_{234}$

Δ_{134}

$$\boxed{\Delta_{126}}$$
$$\nearrow \Delta_{125}$$
$$\boxed{\Delta_{156}} \qquad \boxed{\Delta_{123}}$$
$$\Delta_{235}$$
$$\boxed{\Delta_{234}}$$
$$\Delta_{235}$$
$$\Delta_{356}$$
$$\boxed{\Delta_{456}} \qquad \boxed{\Delta_{345}}$$
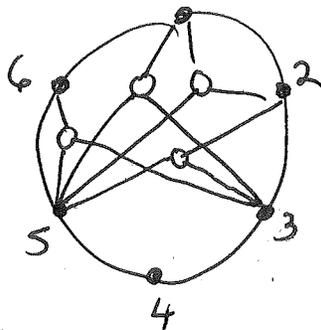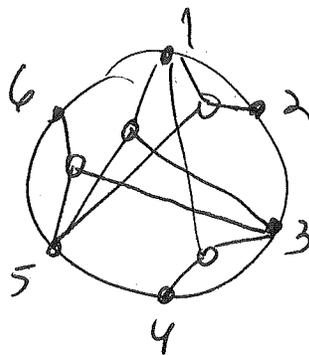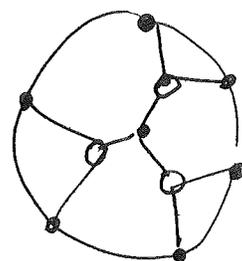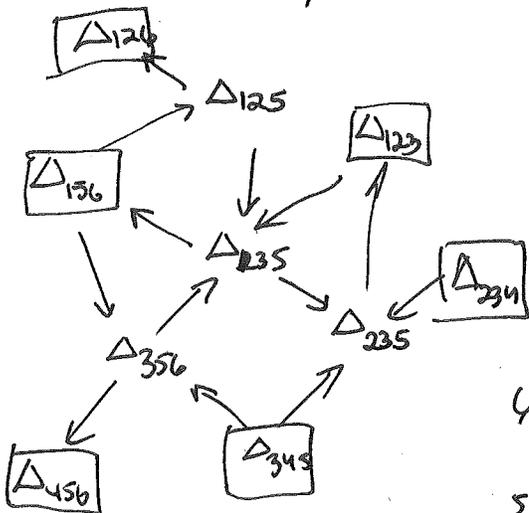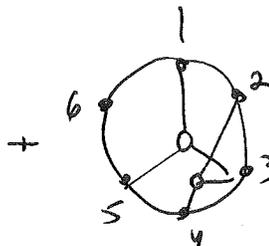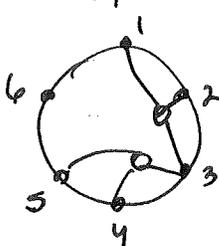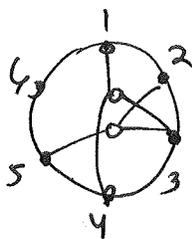
$\boxed{m}$ = frozen variable

Scott's thesis also sets up a cluster algebra
structure on coord ring for any $Gr(k,n)$.

PROBLEM: Build such a "web" theory for $k=4$.
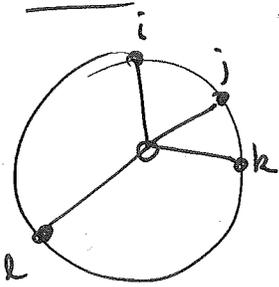
A student of Kuperberg built such a candidate with these local building blocks
Kim

  ,   and rules like

  =

$$\left(\overset{?}{=} \quad \text{} \right)$$
more canonically?

  +

  +
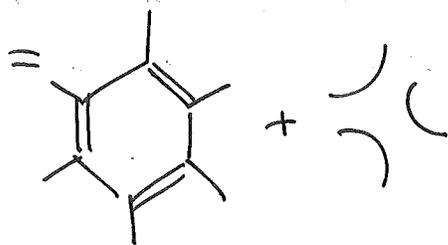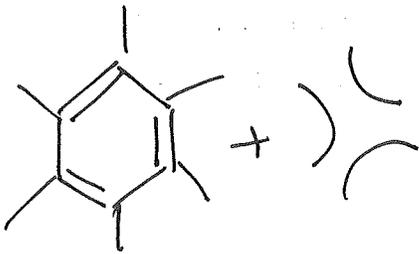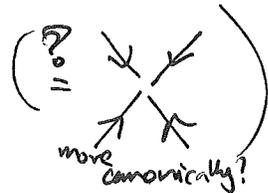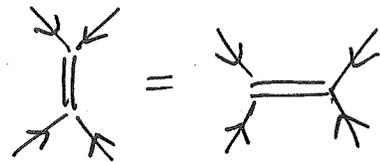
Who to pick in a basis?

Q: Can one experiment with all the cluster variable one gets, to produce
better pictures that suggest a web-like basis.