C.P.S. 9/19/2014 Joel Lewis

Given $D \subset [n] \times [n]$,

we can ask how many $M \in GL_n(\mathbb{F}_q)$ have support avoiding D?

e.g. $n=4$ $\quad D = \{(1,4),(3,2)\}$

$$\#\left\{ M = \begin{array}{|c|c|c|c|} \hline * & * & * & 0 \\ \hline * & * & * & * \\ \hline * & 0 & * & * \\ \hline * & * & * & * \\ \hline \end{array} \in GL_4(\mathbb{F}_q) \right\} = ? = \underbrace{(q-1)^4}_{\substack{\text{always there} \\ \text{because of} \\ (\mathbb{F}_q^\times)^n\text{-action} \\ \text{scaling rows}}} q^6 (q^4 + 3q^3 + 5q^2 + 4q + 1)$$

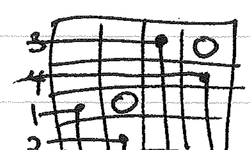
Observation:

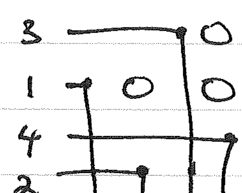
It's not always in $(q-1)^n \mathbb{N}(q)$

e.g. $\begin{array}{|c|c|c|} \hline 0 & * & * \\ \hline * & 0 & * \\ \hline * & * & 0 \\ \hline \end{array} \qquad (q^2 + 2q - 1)(q-1)^3 q$

In fact, it's not necessarily in $\mathbb{C}[q]$, starting at $n=7$.

CONJ: When D is the diagram of a permutation w, the answer does lie in $(q-1)^n \mathbb{N}[q]$

e.g. 3412 $\rightsquigarrow$ 

3142 $\rightsquigarrow$ 

True if w avoids 4231, ——, ——, 351624

and the factor in $\mathbb{N}[q]$ is essentially the Poincaré polynomial of X_w

i.e. rank gen. fn of $[e, w]$