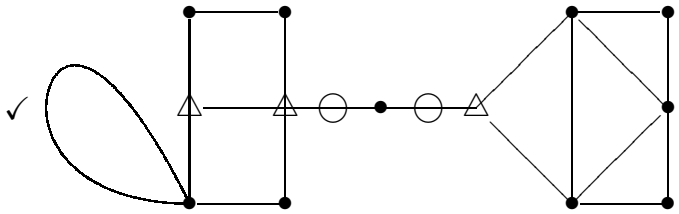
Math 1001 Quiz 1 Solutions

1. (8 points) Place the following labels on the graph below.

   (a) (3 points) Place a △ on each vertex of odd degree.

   (b) (3 points) Place a ◯ on each bridge.

   (c) (2 points) Place a ✓ next to each loop.



2. (10 points, 2 each) True or false. Circle the correct answer, no justification.

   FALSE A graph can have exactly 3 vertices of odd degree.

   FALSE When you are counting the degree of a vertex, loops only count for 1.

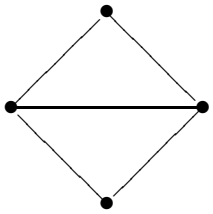
   TRUE If a graph has more than 2 vertices of odd degree, it has no Euler path.

   TRUE Adding loops has no effect on whether a graph has an Euler circuit.

   FALSE Every graph with no vertices of odd degree has an Euler circuit.
   (It has to be connected!)

3. (7 points) Only 5 of the original 7 bridges of Königsberg are still in place. There are still 2 banks (top and bottom) and 2 islands (left and right), but now each bank has exactly one bridge to each island and the islands have one bridge between them.

   (a) (3 points) Draw a graph representing this situation.



   (b) (2 points) Describe an Euler path in this graph.
   One such path: Left to top to right to bottom to left to right.

(c) (2 points) Give a short reason why this Euler path is impractical for tourists.
One such reason: The Euler path has to start on one island and end on another.