1. (4 points, 1 point each) True or false.

   TRUE In a vote with four players, there are $4! = 24$ different sequential coalitions.

   FALSE The quota in a weighted voting system can be less than 50% of the total votes.

   TRUE In the weighted voting system $[9; 10, 3, 3, 2]$, player 1 (P_1) is a dictator.

   FALSE In the weighted voting system $[100; 30, 30, 20, 20, 10]$, the coalition $\{P_1, P_3, P_4, P_5\}$ is a winning coalition.

2. (3 points) List all the winning coalitions in the voting system $[5; 3, 3, 2]$.

   The winning coalitions are $\{P_1, P_2\}, \{P_1, P_3\}, \{P_2, P_3\}$, and $\{P_1, P_2, P_3\}$.

3. (3 points) Suppose that we have a voting system with 3 players, and we know that the winning coalitions in our voting system are $\{P_1, P_2, P_3\}$, $\{P_1, P_2\}$, and $\{P_1, P_3\}$. Find the Banzhaf power indices of these three players.

   In the first coalition, P_1 is critical because, when removed, it is no longer one of these three winning coalitions. However, P_2 and P_3 are not critical because, when removed, they leave winning coalitions.

   All players are critical in the second two coalitions, because there are no winning coalitions with only 1 player.

   Therefore, P_1 is critical 3 times, while P_2 is critical once and P_3 is critical once. Therefore, P_1 has power index $3/5 = 60\%$ and P_2 and P_3 both have power index $1/5 = 20\%$.

4. (3 points) Suppose our voting system is $[11; 5, 4, 3, 3, 2]$. In the sequential coalition $\langle P_1, P_3, P_5, P_2, P_4\rangle$, who is the pivotal player?

   P_1 has 5 votes, which is not enough. P_1 and P_3 have 8 votes, which is not enough. P_1, P_3, and P_5 have 10 votes, which is not enough. P_1, P_3, P_5, and P_2 have 14 votes, which is enough, so P_2 is the critical player.

5. (3 points) Suppose we have a voting system with 4 players, and we found that P_1 is pivotal in 12 sequential coalitions, P_2 is pivotal in 8 sequential coalitions, and P_3 and P_4 are pivotal in the same number of coalitions each. Find the Shapley-Shubik power indices of all 4 players. (You can leave your answer as a fraction if you prefer.)

   There are $4! = 24$ sequential coalitions. If P_1 is pivotal in 12 and P_2 is pivotal in 8, then P_3 and P_4 are pivotal equally in the remaining $24 - 12 - 8 = 4$ sequential coalitions. Therefore, P_3 and P_4 are pivotal in 2 sequential coalitions each.

   As a result, P_1 has power index $12/24 = 1/2$, P_2 has power index $8/24 = 1/3$, and P_3 and P_4 have power indices $2/24 = 1/12$.