

Math 1001 Quiz 7 Solutions

1. (8 points, 2 points each) True or false.

FALSE The Koch snowflake encloses an infinitely large area.

TRUE The Koch snowflake has an infinitely long boundary.

TRUE After each step in constructing the Sierpinski gasket, there are three times as many triangles as there were before.

TRUE When playing the “chaos game,” at each stage you roll a die and move *halfway* from your current position to whatever corner corresponds to the die roll.

The next two questions both refer to this construction, called the Cantor set.

Suppose we start with a line. At step 1, we cut out the middle $\frac{1}{3}$ of the line, leaving two smaller lines. We keep repeating this: at each step, we cut out the middle $\frac{1}{3}$ of all the remaining lines. Here is a picture of the first few steps:



2. (3 points) Find a general formula for the number of lines left after step n .

After step 1, there are 2. After step 2, there are 4. After step 3, there are 8. The general formula is that there are 2^n lines left after step n .

3. (3 points) If the line starts with length 1, find a general formula for the length left after step n .

Each step cuts things down to $2/3$ of the original length. After step 1, we have length $2/3$. After step 2, they are length $4/9$. The general formula is that after step n the remaining length is $(2/3)^n$.

4. (3 points) Multiply these two complex numbers.

$$(1 + 2i) \times (5 + 2i) = 1 + 2i + 10i + 4i^2 = 1 + 12i - 4 = -3 + 12i$$

5. (3 points) What does the infinite sum

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$$

add up to?

This is a geometric series where each number is $1/3$ of the previous. Therefore, it adds up to $\frac{1}{1-1/3} = \frac{1}{2/3} = 3/2 = 1.5$.