1. True/False (24 points, 2 each). No partial credit.

   (a) **(TRUE)** If a graph has an Euler path or circuit, Fleury's algorithm always finds it.

   (b) **(FALSE)** The cheapest-link algorithm always finds an optimal solution to the traveling salesman problem.

   (c) **(TRUE)** Kruskal's algorithm always finds a minimal spanning tree in a graph.

   (d) **(TRUE)** A tree with 10 edges must have 11 vertices.

   (e) **(TRUE)** A connected graph with 11 vertices and 10 edges must be a tree.

   (f) **(TRUE)** A complete graph with 5 vertices must have 10 edges.

   (g) **(TRUE)** A complete graph with 11 vertices must have a Hamilton circuit.

   (h) **(TRUE)** Any connected graph can be Eulerized.

   (i) **(TRUE)** If a graph has a bridge, then it cannot have an Euler circuit.

   (j) **(TRUE)** When using the nearest-neighbor algorithm, the Hamilton circuit you get depends on the vertex you start at.

   (k) **(FALSE)** If three points sit in a triangle with angles 36°, 104°, and 40°, then the shortest network between these points is a minimal spanning tree.

   (l) **(FALSE)** When Eulerizing or semi-Eulerizing a graph, you are allowed to add edges between any pair of vertices you want.

2. Multiple choice (9 points). No partial credit.

   (a) (2 points) Suppose I have a connected graph with 7 vertices, of degrees 1, 2, 4, 4, 6, 8, and 9. Does the graph have an Euler path or Euler circuit?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

   The correct answer is B.

   (b) (2 points) What if instead the graph had vertices of degrees 1, 2, 2, 2, 3, 3, and 3?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

   The correct answer is C.

   (c) (2 points) What if instead the graph had vertices of degrees 2, 4, 6, 8, 10, and 20?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

   The correct answer is A.

(d) (3 points) Let's change the question slightly. Suppose a connected graph has 4 vertices of degrees 3, 3, 3, and 3. Does it have a **Hamilton** path or circuit?

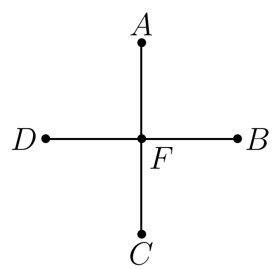
A	It has a Hamilton circuit	B	It has a Hamilton path, but no Hamilton circuit
C	It has neither a Hamilton path nor a Hamilton circuit	D	Not enough information to tell

The correct answer is D. There is a graph like this that does have a Hamilton circuit and another graph that doesn't have a Hamilton path.

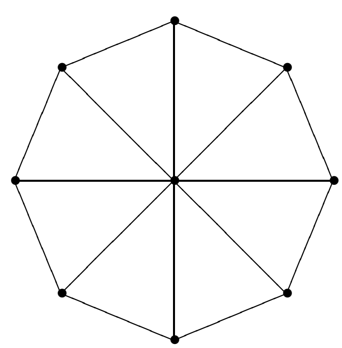
3. (10 points) Draw graphs to represent the following situations.

(a) (3 points) You have an interstate which starts at one city and ends at another, hitting 5 more cities in between.

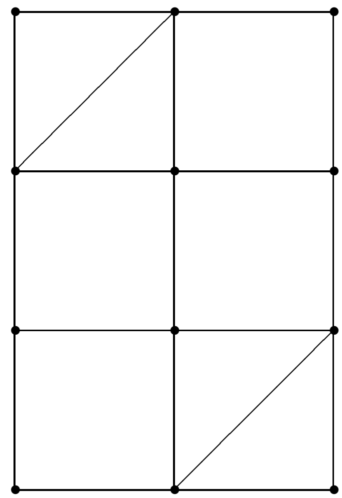
(b) (3 points) You have 4 computers A, B, C, and D that are not connected to each other, but each is connected to the main fileserver F.



(c) (4 points) You have a round building with a large central room and 8 rooms in a circle around the outside, and these rooms are connected by doorways. The central room has 8 doorways connecting it to the outside rooms. Each outside room has 3 doorways: one connecting it to the central room and 2 other doorways connecting it to neighboring rooms in the circle.

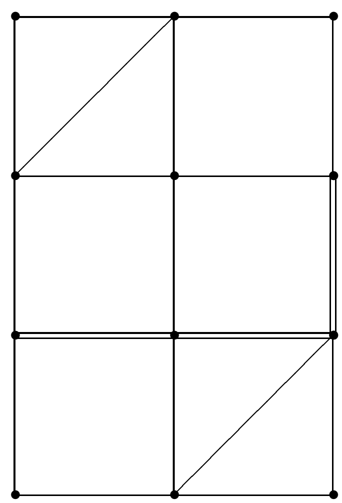


4. (9 points) The following questions all refer to this graph:



   (a) (5 points) Eulerize the graph above using as few edges as possible and explain what you've done.

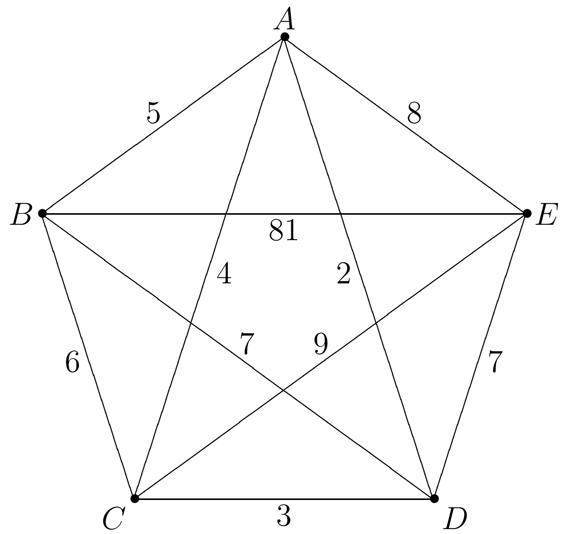
   We need to add edges to get rid of the two odd vertices. We need to add at least 3 edges. One way to do this is as following:



   (b) (4 points) Explain what you would have done if you were asked to semi-Eulerize the graph instead of Eulerizing it.

   Semi-Eulerizing is adding enough edges so that we can have an Euler path. Since the original graph had only two odd vertices, it already had an Euler path, and so we wouldn't need to do anything.

5. (14 points) The following questions refer all refer to this weighted graph:



(a) (5 points) Use the nearest-neighbor algorithm starting at E to find a Hamilton circuit. Give your answer as a list of vertices in order.

From E, the nearest neighbor is D at cost 7. From D, the next nearest-neighbor is A as cost 2. From A, the nearest is C at cost 4. From C, we have to move to B because that is the only vertex left that we haven't visited, and then we go back to E. So the answer is: E, D, A, C, B, E.

(b) (6 points) Use the cheapest-link algorithm to find a Hamilton circuit. Give your answer as a list of vertices in order, starting at A.

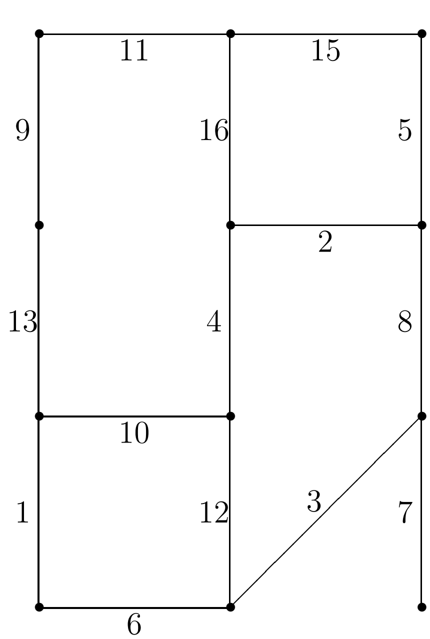
The cheapest link is AD at cost 2. The next is CD at cost 3. AC is next, but it makes a circuit, so we skip it. AB is next at cost 5. BC, BD, DE, and AE are the next links in order of cost, but they all either make circuits or 3-point intersections so we skip them. CE gets added next at cost 9, and finally BE at cost 81. So the final circuit is: E, B, A, D, C, E or E, C, D, A, B, E depending on which order you go around.

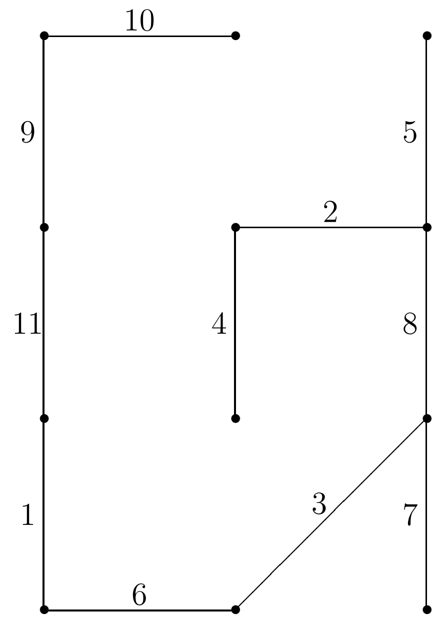
(c) (3 points) Without calculating the cost of any more Hamilton circuits, can you tell if either of these answers is optimal? Why or why not?

Neither of these are minimal, because they were both forced to include the very costly edge BE of cost 81, which outweighs all the other edges by a large margin.

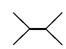
6. (14 points) Almost done.

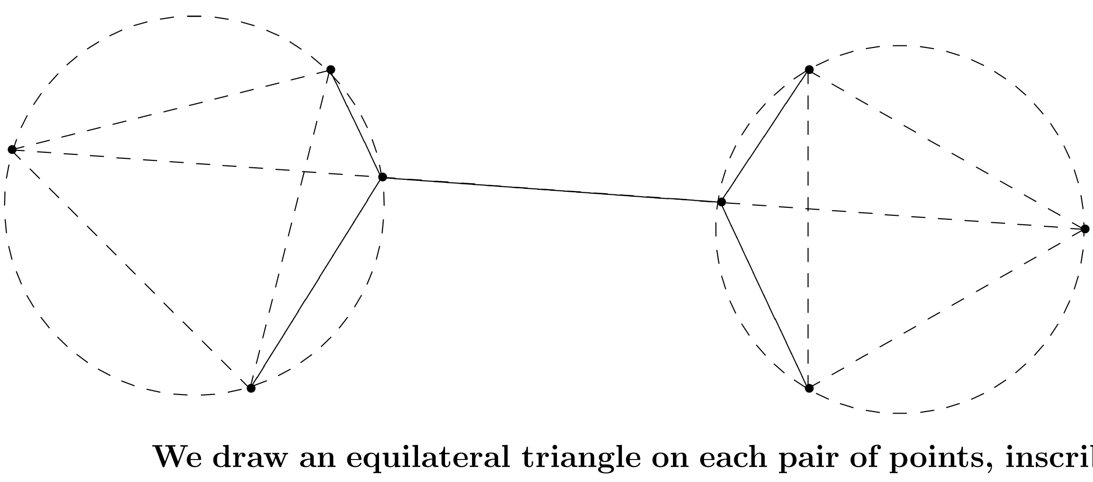
   (a) (8 points) Using Kruskal's algorithm, find a minimal spanning tree for this weighted graph. (Do not calculate the total weight/cost.)





   The answer is at right. The numbers on the edges indicate what order they are added in (cheapest to most expensive).

   (b) (6 points) Use the version of Toricelli's construction from class to construct a network between these 4 points, with 2 Steiner points, that looks roughly like this: 
   Briefly explain your steps. Complete accuracy in drawing is not necessary, but make sure to mark your final network clearly.



   We draw an equilateral triangle on each pair of points, inscribe a circle around each of these, draw a line between opposite corners and mark the points where this line intersects the circles as Steiner points.