Math 1001
Fall 2007
Test 1
10/5/07
Time allowed: 50 minutes

Name: ______________________

Student ID: __________________

Section: 10:10 11:15 12:20

This exam includes 7 pages, including this one and a sheet for scratch work. There are a total of 6 questions on the exam, each with multiple parts.

This test is closed book. You are not allowed to consult the text or any notes you have. Scientific calculators are allowed, but not graphing calculators or any other calculators with more functions.

Show your work. Except where specifically indicated, partial credit can be awarded for work shown on various problems. An incorrect answer with no supporting work will receive little to no credit. Make it clear what your final answer to each question is.

Note that there are questions printed on both sides of each page!

Problem	Total possible	Score
1	24	
2	9	
3	10	
4	9	
5	14	
6	14	
TOTAL	80	

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1. True/False (24 points, 2 each). No partial credit.

_____ (a) If a graph has an Euler path or circuit, Fleury's algorithm always finds it.

_____ (b) The cheapest-link algorithm always finds an optimal solution to the traveling salesman problem.

_____ (c) Kruskal's algorithm always finds a minimal spanning tree in a graph.

_____ (d) A tree with 10 edges must have 11 vertices.

_____ (e) A connected graph with 11 vertices and 10 edges must be a tree.

_____ (f) A complete graph with 5 vertices must have 10 edges.

_____ (g) A complete graph with 11 vertices must have a Hamilton circuit.

_____ (h) Any connected graph can be Eulerized.

_____ (i) If a graph has a bridge, then it cannot have an Euler circuit.

_____ (j) When using the nearest-neighbor algorithm, the Hamilton circuit you get depends on the vertex you start at.

_____ (k) If three points sit in a triangle with angles 36°, 104°, and 40°, then the shortest network between these points is a minimal spanning tree.

_____ (l) When Eulerizing or semi-Eulerizing a graph, you are allowed to add edges between any pair of vertices you want.

2. Multiple choice (9 points). No partial credit.

_____ (a) (2 points) Suppose I have a connected graph with 7 vertices, of degrees 1, 2, 4, 4, 6, 8, and 9. Does the graph have an Euler path or Euler circuit?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

_____ (b) (2 points) What if instead the graph had vertices of degrees 1, 2, 2, 2, 3, 3, and 3?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

_____ (c) (2 points) What if instead the graph had vertices of degrees 2, 4, 6, 8, 10, and 20?

A	It has an Euler circuit	B	It has an Euler path, but no Euler circuit
C	It has neither an Euler path nor an Euler circuit	D	Not enough information to tell

_____ (d) (3 points) Let's change the question slightly. Suppose a connected graph has 4 vertices of degrees 3, 3, 3, and 3. Does it have a **Hamilton** path or circuit?

A	It has a Hamilton circuit	B	It has a Hamilton path, but no Hamilton circuit
C	It has neither a Hamilton path nor a Hamilton circuit	D	Not enough information to tell

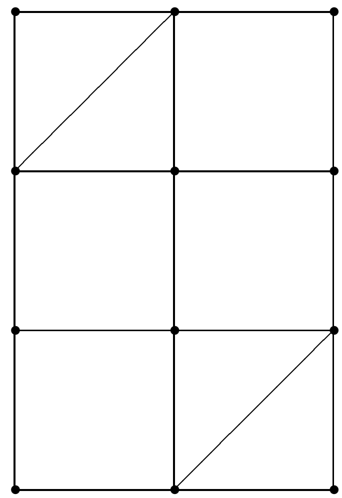
3. (10 points) Draw graphs to represent the following situations.

   (a) (3 points) You have an interstate which starts at one city and ends at another, hitting 5 more cities in between.

   (b) (3 points) You have 4 computers A, B, C, and D that are not connected to each other, but each is connected to the main fileserver F.

   (c) (4 points) You have a round building with a large central room and 8 rooms in a circle around the outside, and these rooms are connected by doorways. The central room has 8 doorways connecting it to the outside rooms. Each outside room has 3 doorways: one connecting it to the central room and 2 other doorways connecting it to neighboring rooms in the circle.

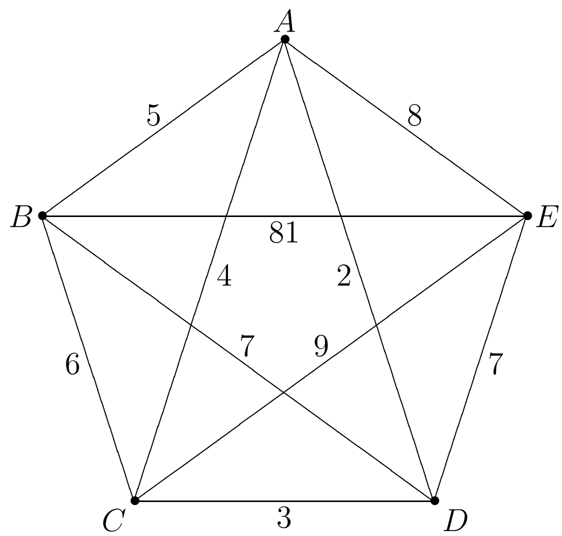
4. (9 points) The following questions all refer to this graph:



(a) (5 points) Eulerize the graph above using as few edges as possible and explain what you've done.

(b) (4 points) Explain what you would have done if you were asked to semi-Eulerize the graph instead of Eulerizing it.

5. (14 points) The following questions refer all refer to this weighted graph:



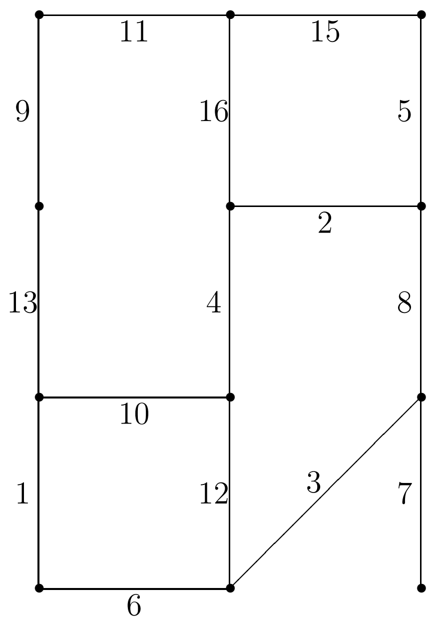
(a) (5 points) Use the nearest-neighbor algorithm starting at E to find a Hamilton circuit. Give your answer as a list of vertices in order.

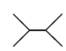
(b) (6 points) Use the cheapest-link algorithm to find a Hamilton circuit. Give your answer as a list of vertices in order, starting at A.

(c) (3 points) Without calculating the cost of any more Hamilton circuits, can you tell if either of these answers is optimal? Why or why not?

6. (14 points) Almost done.

   (a) (8 points) Using Kruskal's algorithm, find a minimal spanning tree for this weighted graph. (Do not calculate the total weight/cost.)



   (b) (6 points) Use the version of Toricelli's construction from class to construct a network between these 4 points, with 2 Steiner points, that looks roughly like this: 
   Briefly explain your steps. Complete accuracy in drawing is not necessary, but make sure to mark your final network clearly.