Math 2374
Practice final exam answers and hints
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1. $\frac{513}{135}$.

2. $(8,-6,6)\cdot(x-2,y-3,z-1)=0$.

3. 8π.

4. (a) $-3+\frac{1}{2}(-3(x-2)^2-2(y-8)^2+2(x-2)(y-8))$. (Note that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are 0 at $(2,8)$ because it is a critical point.)

   (b) Local max at $(2,8)$. ($D=5$ and $f_{xx}=-3$.)

5. 0. By Stokes' Theorem, the line integral along each C_i is equal to the surface integral of $\nabla\times\mathbf{F}$ on the surface bounded by C_i, and $\nabla\times\mathbf{F}=\mathbf{0}$.

6. 6. ($\operatorname{div}\mathbf{F}=3$, and $3\cdot\mathrm{Vol}(W)=6$.)

7. (a) 0.6. (Use the chain rule.)

   (b) 0.18. (Find the directional derivative.)

8. $-\frac{1}{6}$. (Use Green's Theorem.)

9. (a)
$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^2 \rho^2\sin^3\phi\,d\rho\,d\phi\,d\theta.$$

   (b) $\frac{32\pi}{9}$. (Use Pythagorean Theorem for the ϕ-integral.)

10. This is the same as showing that the integral along C_1 plus the integral along the opposite orientation of C_2 is equal to 0. But this is the boundary of the cylinder S with outward-pointing normal, parametrized by $\Phi(\theta,z)=(\cos\theta,\sin\theta,z)$, for $0\le\theta\le2\pi$ and $1\le z\le2$. By Stokes' Theorem, the sum of line integrals equals
$$\iint_S(\nabla\times\mathbf{F})\cdot d\mathbf{S}.$$
Check that the integrand here is 0.