Study guide for the first exam

Math 2374, Fall 2006

1. Basic vector material (Chapter 1)

   (a) Comments: the initial sections of this course are background material for the rest of the course. The following may help you organize your studying of the diverse topics.

   (b) Key items for exam 1

      i. Computing 2×2 and 3×3 determinants: although these become more important later in the course, you can use them now to help you memorize the cross product.

      ii. Dot products and cross products: these are used all the time, throughout the whole course. Make sure you understand and can compute them.

      iii. Parametrizations of lines and equations of planes: these form an important basis of the course. A good understanding of them will be important.

      iv. Vectors in $\mathbf{R}^n$. Be able to find magnitudes of vectors.

      v. Matrices. Multiply matrices times vectors, matrices times matrices.

   (c) Notes

      i. We don't cover cylindrical and spherical coordinates (Section 1.4) until later in the course.

   (d) Sample book problems: 1.3 #15(d), #16(b), #26, #30, 1.5 #8

2. Functions and graphing (Section 2.1)

   (a) Three-dimensional graphing: the only graphs in three dimensions we might ask you to sketch are quadric surfaces, planes, cylinders, lines, as well as portions or combinations of these.

   (b) Level sets: level curves for functions of two variables and level surfaces for functions of three variables. Be able to sketch a few level curves as in the homework.

   (c) Sample book problems: 2.1 #1(a), #2(b), #5

3. Derivatives (a big focus of the exam)

   (a) Partial derivatives (Section 2.3)

      i. Key items: understand and compute partial derivatives

      ii. Methods: limit definition, one-variable calculus techniques.

      iii. Sample book problems: 2.3 #2(b), #3(b)

(b) The derivative (Section 2.3)

   i. Key idea 1: the derivative is represented by the matrix of partial derivatives
   ii. Key idea 2: use the derivative to write a linear approximation of a function f near a point $\mathbf{a}$.
   iii. Key idea 3: a function being differentiable at a point means it is nearly linear around that point.
   iv. Key idea 4: if the partial derivatives in the matrix of partial derivatives are continuous at a point, then the function is differentiable.
   v. Note that for $f : \mathbf{R}^2 \to \mathbf{R}$, the linear approximation is the tangent plane.
   vi. For a scalar-valued function, the derivative can be written as vector (the gradient).
   vii. Sample book problems: 2.3 #6(b). #12(b), #13(c), #14(c)

(c) Introduction to paths and curves (Section 2.4)

   i. Kknow that a curve can be parametrized by a function $\mathbf{c}(t)$, that $\mathbf{c}'(t)$ is the velocity of an object with position $\mathbf{c}(t)$, and $\mathbf{c}'(t)$ is tangent to the path.
   ii. Be able to compute a tangent line to a curve.
   iii. Sample book problems: 2.4 #15, #17

(d) The chain rule (Section 2.5)

   i. Key idea: The chain rule gives the derivative of a composition of functions.
   ii. Key formula: $D(f \circ g)(\mathbf{a}) = Df(g(\mathbf{a}))Dg(\mathbf{a})$
   iii. Note: Formulas for partial derivatives can be derived from above formula, but be careful to evaluate partial derivatives of f at the point $g(\mathbf{a})$.
   iv. Sample book problems: 2.5 #2(f), #5(b), #9, #13

4. The gradient and the directional derivative (Section 2.6)
(we assume functions are differentiable)

   (a) The gradient

      i. Key idea: for scalar-valued function f, the gradient ∇f is like the matrix of partial derivatives Df, except that the gradient is a vector rather than a matrix.
      ii. The gradient is a vector whose magnitude and direction have physical meaning.
         A. The gradient points in the direction where f increases most rapidly.
         B. The magnitude of the gradient indicates the rate of change in f in that direction.
      iii. Since the gradient is perpendicular to level sets of f, you can use the gradient to find tangent tangent planes to surfaces.
      iv. Sample book problems: 2.6 #4(c), #7(c)

   (b) The directional derivative

i. Key idea: the directional derivative is a generalization of the partial derivative. The directional derivative $D_{\mathbf{u}}f$ gives the rate of change of f in the direction specified by $\mathbf{u}$ ($D_{\mathbf{u}}f$ represents slope in that direction).

ii. Important formula: $D_{\mathbf{u}}f(\mathbf{a}) = \nabla f(\mathbf{a}) \cdot \mathbf{u}$ (alternatively $D_{\mathbf{u}}f(\mathbf{a}) = \|\nabla f(\mathbf{a})\| \cos\theta$)

iii. Don't forget: $\mathbf{u}$ must be a **unit vector**.

iv. Although the gradient is a vector, the directional derivative is a scalar.

v. If $\mathbf{u}$ is perpendicular to the gradient, then $D_{\mathbf{u}}f = 0$. If $\mathbf{u}$ points in the same direction as the gradient, then $D_{\mathbf{u}}f = \|\nabla f\|$. If $\mathbf{u}$ points in the opposite direction of the gradient, then $D_{\mathbf{u}}f = -\|\nabla f\|$.

vi. Sample book problems: 2.6 #3(b), #20