Math 5286H

Problem Set 1

Due on **Wednesday, March 1.**

True/false. Correct answers are 2 points, incorrect worth 0 points, "I don't know" worth 1 point.

________ If F is a field, then the fraction field of F is isomorphic to F.

________ The ring $\mathbb{Z}/6$ has a fraction field.

________ The ring $\mathbb{Z}/(nm)$ is isomorphic to $\mathbb{Z}/n \times \mathbb{Z}/m$ *only* when n and m are relatively prime.

________ There is an isomorphism of rings $\mathbb{C}[x,y]/(y-x^2-x, y+x^2+x) \cong \mathbb{C}\times\mathbb{C}$.

________ The ideal $(x^2+y^2, x+1, y^2+y+1)$ is a maximal ideal of $\mathbb{C}[x,y]$.

Short answer. 5 points each for a correct answer.

1. The intersection of the ideals (x) and (y) of $\mathbb{C}[x,y]$ is the ideal ________.
2. The elements ________ and ________ of the ring $\mathbb{Z}[i]$ are the only solutions to the equation $x^2+6x+8=0$.

Long form. 10 points.

1. Show that the ring
$$\mathbb{Z}[x]/(x^2-2)$$
is an integral domain, and that the fraction field is isomorphic to the ring
$$\mathbb{Q}[x]/(x^2-2).$$