Math 5286H

Problem Set 3

Due on **Monday, March 21.**

True/false. Correct answers are 2 points, incorrect worth 0 points, "I don't know" worth 1 point.

________ The ring $\mathbb{Q}[x]/(x^3 - 2x + 1)$ is a field.

________ The polynomial $x^5 + yx^2 - y$ is irreducible in $\mathbb{C}[x, y]$.

________ The polynomial $x^5 - 144x + 96$ is irreducible in $\mathbb{Q}[x]$.

________ The ring $\mathbb{C} \times \mathbb{C}$ is a principal ideal domain.

________ The ring $\mathbb{Q}[x, y]/(ax + by, cx + dy)$ is finite-dimensional as a vector space over $\mathbb{Q}$ if and only if the determinant $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is nonzero.

Short answer. 5 points each for a correct answer.

1. The factorization of the element $20x^4 - 80$ into irreducibles in $\mathbb{Z}[x]$ is ________.
2. The number of prime ideals in the ring $\mathbb{C}[x]/(x^2(x-1)^3(x-2)(x-4))$ is ________.

Long form. 10 points.

1. Show that, for any integer x, the integer $x^3 + x^2 - 2x - 1$ is never divisible by 3 or 5. (Hint: Modular arithmetic.)