Math 8253, Fall 2015
Homework 1
Due in-class on **Monday, September 21**

1. Determine all the maximal prime ideals of the ring $\mathbb{R}[x]$, and the effect of the map

$$\{\text{prime ideals of } \mathbb{C}[x]\} \to \{\text{prime ideals of } \mathbb{R}[x]\}$$

given by $\mathfrak{P} \mapsto \mathfrak{P} \cap \mathbb{R}[x]$.

2. Determine the irreducible components of the closed algebraic subset defined by the equations

$$x^2 - x = y^2 - y = xy - y = xy - x = 0$$

in two variables.

3. From commutative algebra: if R is a unique factorization domain, then so is the polynomial ring $R[x]$. State the classification of irreducible elements of $R[x]$, and use it to show that the closed algebraic set defined by $y^2 = x^3 + ax^2 + bx + c$ is irreducible for any $a, b, c \in \mathbb{C}$. (Be warned that "irreducible" means two distinct things in this problem.)

4. Show that the algebraic set of (x, y, z) such that

$$xz = y^2, x^3 = yz, z^2 = x^2 y$$

is irreducible, and that the solutions are all of the form (t^3, t^4, t^5). Show the first two of these equations define an algebraic subset with two irreducible components, and describe the other component.